

# Negation, material incompatibilities and inferential thickness: a Brandomian take on Middle Wittgenstein

MARCOS SILVA

## §1. Introduction

**B**RANDOM'S WORK (1994, 2001, 2008) IS WELL KNOWN for motivating the transition in contemporary discussions about semantics from a referentialist account of meaning to an inferentialist account. According to him, meaning is given by the rules which govern concepts and not by referring some linguistic expressions to extralinguistic elements. Since no propositions are logically isolated, they are conceptually and materially dense and should be thought of as logically organised in terms of implications and exclusions. As a result, they display the property of being "inferentially thick", as some inferentialists maintain (for instance, Stekeler-Weithofer 2013). Brandom himself holds that "inferentialism about conceptual contents means taking them to be identified and individuated by their inferential roles" (2001, p. 61). On this view, conceptual content is determined by the function it plays in inferential chains; the content of each concept is articulated by and encapsulated in its inferential relations to *other* concepts. Accordingly, in an inferentialist account of conceptual content, every concept is associated with *many* concepts. Propositional content, then, states Brandom, "must come in packages (though it does not yet follow that they must come in just one great big one)" (2001, pp. 15–16).

In this paper, I defend the notion that Middle Wittgenstein's concept of *Satzsysteme*<sup>1</sup> in his return to philosophy in 1929 is a forerunner of an inferentialist

<sup>1</sup> Since there is no adequate translation for Satz into English, I will use the German original expression Satzsystem in this text

take on semantics, especially concerning the role of negation. Concerning Wittgenstein's return to philosophy, I argue that if we have elementary propositions, they should all be inserted in several different systems which are logically organised through material exclusions, wherein negation "explodes" in several if not in infinite alternatives.

The usual formal non-material way to understand negation is as contradictoriness. It is related to the idea of understanding  $\neg A$  as the contradictory of  $A$ . On this view, the relationship of contradiction may be defined in terms of certain logical laws, such as the principle of excluded middle and the principle of non-contradiction. However, to operate and reason with propositions in Middle Wittgenstein's *Satzsysteme*, we should know the whole system in which the proposition is inserted to understand which combinations are *allowed* and which are *prohibited*. The main lines of the abandonment of the Tractarian logical atomism in Wittgenstein's middle period are compatible with several inferentialist points, as Brandom for example presents:

Inferentialism of any sort is committed to a certain kind of semantic *holism*, as opposed to the *atomism* that often goes hand in hand with commitment to a representationalist order of semantic explanation. For if the conceptual content expressed by each sentence or word is understood as essentially consisting in its inferential relations (broadly construed) or articulated by its inferential relations (narrowly construed), then one must grasp many such contents in order to grasp any" (Brandom 2001, p. 29).

Concerning meaning, Wittgenstein's return to philosophy in 1929 is commonly associated with some form of verificationism in relation to Vienna Circle discussions (Engelmann 2013, Hacker 1989). Wrigley (1989), for instance, considers Wittgenstein's verificationism about meaning in the beginning of the 1930s to be "extreme", "radical" and "dramatic". In fact, Wittgenstein seems to defend the idea that to understand the sense of a proposition means to understand its method of verification. Exploring its contrapositive, this principle implies that to fail to grasp the method of verification of a proposition means to fail to understand the proposition. By any measure, this fact seems to justify, for instance, Wittgenstein's insistence on grasping "eine Methode des Findens, Suchens" ["a method of finding, searching"] in order to understand what a proposition means (*PB* § 43<sup>2</sup>). Accordingly, we need to

<sup>2</sup> Here I use *PB* for *Philosophische Bemerkungen*, *SRLF* for "Some Remarks of Logical Forms" and *WWK* for *Wittgenstein und der Wiener Kreis*. All decimals present in this text come from the number of passages in the *Tractatus Logico-Philosophicus* (*TLP*, or *Tractatus*).

*know how* to determine whether such-and-such is the case to understand the propositional sense of a linguistic expression. Note that Middle Wittgenstein's discussion is not just about truth and how we refer to things but is about sense and how some sort of evidence may constrain our understanding. One natural question against this background is "Does his return to philosophy make epistemology be related to semantics?" If so, it would be controversial for Wittgenstein, as in *TLP* he considered epistemology a kind of psychology (4.1121).

This paper is divided into three parts to deal with the problems presented above. Section 2 discusses different forms of verificationism and critically addresses Wrigley's controversial hypothesis (1989) that Wittgenstein's *Tractatus* already contains some form of verificationism. Section 3 examines the so-called colour-exclusion problem as a motivation for Middle Wittgenstein's verificationism and its relations to an inferentialist view of meaning. The last section presents some problems concerning the principle of excluded middle (*PEM*) in the context of Middle Wittgenstein's *Satzsysteme*.

## § 2. On different forms of verificationism

The first question guiding us to understand the kind of semantics which Wittgenstein was envisaging at the beginning of the 1930s should then be, "Why did (Middle) Wittgenstein engage in (explicit) forms of verificationism about meaning at that moment"? The answer proposed here should motivate, in semantics, the transition from a referentialist semantic tradition to an inferentialist. It is important to show why his *Satzsysteme* have the crucial inferentialist feature of being "inferentially thick". In this account, the meaning of a linguistic expression has to be identified with its inferential relationship to many other expressions. Here I am looking for both some internal (conceptual) and some historical reasons for this transition. Some authors (Newen 1994, Marion 1998, Kienzler 1997) point to Brouwer's influence on Wittgenstein's return to philosophy to explain his turn to a constructivist approach to semantics. Others, like Wrigley (1989), Frascolla (2017) and Hacker (1986), try to find in the *Tractatus* forms of verificationism.

I hold that if one wants to put emphasis on verificationism, it should be seen in more differentiated light; there are some inferentialist lessons to be learned in the context of the so-called colour-exclusion problem, particularly lessons about the role of negation in taxonomic systems that are organised in terms of material incompatibilities. These logical features are decisive in understanding the "inferential density or thickness" which governs Wittgenstein's *Satzsysteme*, when seen in the light of Brandom's variant of inferentialism (1994, 2001, 2007).

According to Porto and Pereira (2003), it is important to draw the distinction between two important forms of verificationism. On the one hand, there is the traditional Vienna Circle version. It is verificationist in a strict sense, requiring that for a claim to be meaningful it must be implied by a finite number of observation sentences. A proposition is considered comparable to reality only if, in principle, it can be (empirically) verified. On the other hand, there is what one might call the “New Verificationism” in the philosophy of language, inspired by Dummett’s intuitionism proposing a semantics of conditions of assertability that explains how we understand linguistic vocabulary.

Following Löf’s “Verificationism then and now” (2013), it is possible to relate both verificationist traditions to Wittgenstein’s verificationism at the beginning of the 1930s. In his paper, Löf addresses the semantics of our language by distinguishing two main approaches. The first is the traditional way of thinking of semantics in terms of laws of proposition, a way which emphasises ontological features in understanding linguistic expressions, such as truth conditions, references, states of affairs, models and facts. The second approach highlights Löf’s laws of judgement, which focus on some epistemological features, such as warrant, evidence and assertability, to determine the meaning of propositions. Löf’s distinction between the two approaches, namely between the one concerned with laws of proposition and the other primarily focusing on laws of judgement, can be used to investigate two semantic interpretations of verificationism, which, also cover the distinction, mentioned above, proposed by Porto and Pereira (2003). Löf (2013) introduces verificationism on the one hand, stipulating an empirical method, which means, all claim should be tested by empirical observation. On this view, the existence of a specific empirical state is the condition for us to recognise that the proposition is true. And on the other hand, verificationism should be fixed by a method of proof, based on the notion of introduction rules, not on any empirical state. On this view, truth and reference are not primitive notions; truth depends on the existence of a proof. The leading question in this context is, “What do we have to know to have *the right* to affirm that  $p$  is true”? Löf then defends the idea that the introduction rules are what must be known to enable the understanding of linguistic expressions. In other words, to understand  $p$  is to understand how the verification of  $p$  is to be done. The proposition stipulates how its proof is to be obtained. To understand  $p$  implies to know how to prove it or, simply stated, *how to verify it*.

Note that Löf’s proposal is a normative reading of verificationism rather than an epistemological and ontological reading. For him, verificationism should render a basis for identifying the *legitimate* parts of discourse (both empirical and non–

empirical). As a result, some expressions should be *prohibited, ruled out*. This normative reading of verificationism is connected with the idea of meaning being learned by the “introduction rules”, since to understand the proposition, we must understand how it stipulates how its proof (or verification) is to be obtained.

The picture of verificationism obtained this far already runs counter to Wrigley’s (1989) hypothesis that Wittgenstein’s *Tractatus* presupposes (or implies) verificationism. In fact, the distinction between two semantic theories, namely between a semantics relying on truth conditions and a semantics based on conditions for the ascription of truth, are at the centre of the discussion of Wittgenstein’s return to philosophy. Following Dummett (1978, 1991), Krikpe (1982) and more recently, Löf (2013), I think we should see Wittgenstein’s verificationism at the beginning of the 1930s as a break with his account of meaning in the *Tractatus* and not assimilate the former to the latter. Wrigley seems not to appreciate sufficiently that the *Tractatus* displays truth–conditions semantics, as it has no evidence–constrained conception of truth. The *Tractatus* has a realist account of propositional content based on the notion of truth conditions (*TLP* 4.41, 4.431, 4.442, 4.45–4.461 and 4.463). There, propositional content should be exhaustively and exclusively determined by its truth conditions. Verification seems to be irrelevant in this scenario. As a result, Tractarian operators provide a *realistic* semantics which is neutral, unlimited and essentially combinatorial. It should be outlined that every combination of elementary propositions is always possible there (Silva 2016a, 2016b). The *Tractatus* is an excellent example of an approach to semantics, by what Löf called laws of proposition, which poses no restriction on the formation of molecular propositions from a pool of legitimate atomic propositions.

However, after some important setbacks, Wittgenstein noted that logical operators should be *topically sensitive* (see *PB* 81–83, Prado Neto 2003, Engelmann 2013, Cuter 2017). We should *test* and *verify* the possibility of propositional formation, since some (complex) propositions are not *allowed*. Some constructions should be *forbidden*. The free distribution of truth values, independent of the existence of empty names, should be restricted, constrained and prohibited in some different conceptually organised systems. Note that here we are already using normative vocabulary to understand how non–neutral logical operators should function, as Löf’s contention about the real nature of verification suggests.

Accordingly, Engelmann makes the following remark about Wittgenstein’s verificationism: “(...) the post–1929 equivalence of a proposition having sense and having a method of verification should be seen as response to the problems related to Ramsey’s objection” (2013, *p.* 27). Ramsey’s objection is this: In 1923, in a

historical review of the *Tractatus*, Ramsey spelled out the so-called colour-exclusion problem. There are some inferentialist lessons to be learned in this context, especially about the inferential role of negation expressing material incompatibilities. Those lessons may take their light from Brandom's inferentialism, as for him, "negation, as a logical connective supporting formally valid inferences, plays the same explicating role with respect to material *incompatibility* relations among judgeable (that is propositional) contents that the *conditional* plays with respect to material *inferential* relations among such contents" (1994, p. 115).

By using Löf's (2013) normative proposal of reading verificationism, it is possible to relate Wittgenstein's problem with expressing conceptual relations between colours to some problems with verificationist remarks which arise in the context of his phenomenological *Satzsysteme*. Some examples of these *Satzsystemes* discussed by Middle Wittgenstein are systems of colours, temperature, volume, hardness, length, weight and height. In all these "phenomenological systems", the principle of excluded middle (*PEM*) does not hold. The reason for this failure of *PEM* amongst them is simple, as we will see in section 4. My point is that sentences which ascribe a colour to a visual point or a degree to an empirical quality are clearly mutually exclusive, for they cannot be true together, *but they are not contradictory, because they can be false together*. This logical feature grounds the material incompatibility in a colour system in particular and in *Satzsysteme* in general.

### § 3. Colour-exclusion problem as a motivation for inferentialism

In 1923, Ramsey spelled out the so-called colour-exclusion problem in his famous review of the *Tractatus*:

It is a principle of Mr. Wittgenstein's, and, if true, is a very important discovery, that every genuine proposition asserts something possible, but not necessary. This follows from his account of a proposition as the expression of agreement and disagreement with truth-possibilities of independent elementary propositions, so that the only necessity is that of tautology, the only impossibility that of contradiction. There is great difficulty in holding this; for Mr. Wittgenstein admits that a point in the visual field *cannot* be both red and blue; and, indeed, otherwise, since he thinks induction has no logical basis, we should have no reason for thinking that we may not come upon a visual point which is both red and blue. Hence he says that "This is both red and blue" is a contradiction. This implies that the apparently simple concepts red, blue (supposing us to mean by those words absolutely specific shades) are really complex and formally incompatible. He tries to show how this may be, by analysing them in terms of vibrations. But even supposing that the physicist thus provides an analysis of what we mean by "red" Mr. Wittgenstein is only reducing the difficulty to that of the *necessary* properties of space, time,

and matter, or the ether. He explicitly makes it depend on the *impossibility* of a particle being in two places at the same time. These necessary properties of space and time are hardly capable of a further reduction of this kind. For example, considering between in point of time as regards my experiences; if B is between A and D and G between B and D, then G must be between A and D; but it is hard to see how this can be a formal tautology. (Ramsey, 1923, p. 273)

Ramsey's criticism motivated the colour system to be targeted as a *logical* problem and not as an epistemological or phenomenological one. The conceptual organisation of a colour system imposes itself as a problem for the Tractarian account of necessity based solely on the notion of formal tautologies. It is against this background that in the *Tractatus*,  $p \sim p$  is the only form of necessary exclusion. However, 6.3751 asserts that point  $a$  in the visual field is blue and the (same) point  $a$  is red (6.3751) is a contradiction. Accordingly, one might ask what is the status of a proposition such as "if  $a$  is green, then it is not red", where  $a$  is a point in the visual field. This proposition seems to make sense and seems to be necessary and *a priori*. But as Ramsey asked in his 1923 review, is it legitimate to call it a tautology?

In 1927, Ramsey returns to the topic of the nature of logic and elaborates on this theme, discussing the nature of the kind of exclusion in a colour system and the difficulties with the Tractarian account of necessity: "it seems to me that formal logic is not concerned with it, but presupposes that all the truth-possibilities of atomic sentences are really possible, or at least treats them as being so. No one could say that the inference from 'this is red' to 'this is not blue' was formally guaranteed like the syllogism" (p. 167).

Ramsey seems to defend a traditional, formal view of logic in the sense that it is not concerned with the conceptual content of propositions. Further, in formal logic, the opposite of a tautology is a contradiction. Contradictory exclusion is more relevant to our discussion than the very nature of formal tautology, because we should ask why " $a$  is green and red", say, must be thought of as a contradiction, as defended in the Tractarian 6.3751. In the *Tractatus*, Wittgenstein does not seem to notice that exclusion in the colour system is difficult to reduce to contradiction and would be a fatal problem for his early project and for his view of logic centred on the notion of tautology. As a matter of fact, it can be argued that there is *no* colour problem in the *Tractatus*. That the colour-exclusion problem is not an explicit problem for the *Tractatus* shows how deeply committed Wittgenstein was to some logicist ideas, such as requiring the complete and unambiguous analysis of propositions. It is assumed on the Tractarian view that any meaningful proposition

has an unequivocal logical form which may not be apparent on its surface<sup>3</sup>. In this sense, the logical form of a proposition is likely to be hidden by the grammatical surface of the proposition and would invariably be very complex. (Consider in this context for example the Fregean explanation of quantification and the Russellian theory of definite descriptions.) The philosopher's task in this tradition is to mobilize available logical resources to find logical forms hidden by the surface of language.

With this scenario in mind, it is easy to see why Wittgenstein at 6.3751 argues that “*a* is red and *a* is blue” should be an instance of a contradiction, that is, it should have the logical form “ $p.\sim p$ ”. If we only have one form of logical necessity, tautology, then we only have a form of logical exclusion, the contradiction. This seems to be what is at stake in these Tractarian passages, starting at 6.37, namely that the only form of logical impossibility is the formal contradiction “ $p.\sim p$ ”. Consequently, the (surprisingly long) 6.3751 passage would play the role of precluding once and for all the possibility that the exclusion amongst colours represents a counterexample to this philosophical conviction.

Note that this problem is not to be thought of as restricted to a colour system, as it can be found in other conceptual systems. Consider the following sentences:

- (1) The table over there is three meters long, and the (same) table over there is four meters long.
- (2) Now it is 25 degrees Celsius, and now it is 26 degrees Celsius.
- (3) Flamengo (a soccer team) lost yesterday, and Flamengo won yesterday.
- (4) The animal over there is a cat, and the (same) animal is a dog.

The oddity in these examples, which seems to encompass the same kind of material exclusion present in a colour system, is that no explicit formal negation, as in 6.3751, is to be found in them either. Moreover, if the conjunction expressed in all these examples is contradictory, the disjunction should be tautological. But none of the following examples is a tautology: “point *a* in the visual field is blue or red”, “the table over there is three meters long or the same table is four meters long”, “the

<sup>3</sup> Note one important significant to Russell in the book: TLP 4.0031. In this passage, Wittgenstein contrasts apparent form (*scheinbare Form*) and real form (*wirkliche Form*), the latter to be discovered by careful logical analysis. It is also important to draw attention to a note written by Wittgenstein in September 1913 which sets the tone for the Tractarian project: “Distrust regarding grammar is the first requirement to philosophize” (1961, p. 93).



animal over there is a cat or is a dog”, “my soccer team won or lost yesterday”. Nor is “now it is 25 degrees Celsius or 26 degrees Celsius” a tautology.

It is crucial to note that the logical pattern these conceptual systems share is the one of contrarities, as shown in the usual material relations based on taxonomy and on some rules. It is also noteworthy that Wittgenstein, when he returned to philosophy, does not refer to these logical patterns as laws, axioms or principles but as rules which are “in some sense tautologies”, or in the case of a negation, as “some sort of contradiction” (*SRLF*, pp. 167, 168).

In this context, it is useful to try to capture the logical phenomenon of contrarities with Tractarian notation. The motivation for this strategy is Wittgenstein’s discussions in 1929 and Von Wright’s work (1996) on modal logics. The general leitmotif here is as follows. If the logical organisation of colours represents a problem for Wittgenstein’s logic, it should represent a problem for his notation too<sup>4</sup>. Here we examine the distinction between contradiction and contrarities, using truth tables, using Wittgenstein’s work and comparing truth tables in 1921 and truth tables in 1929.

In 1921, we have the following truth table for conjunction:

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

The above truth table may be interpreted in the following unproblematic way using natural language:

John is scientist	John is logician	John is scientist and John is logician
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<sup>4</sup> For a discussion on whether Wittgenstein could be seen as the father of truth tables, see Silva 2016c.

T	T	T
T	F	F
F	T	F
F	F	F

Applying the 6.3751's way-out to colour-exclusion problem for the same kind of truth table, we would have something like this, where the last column shows the conjunction as a contradiction:

a is red	a is blue	a is red and a is blue
T	T	F
T	F	F
F	T	F
F	F	F

As any complex sentences, conjunctions are formed by the operation of joint negation in the *Tractatus*. But how does the *N*-Operator work in the case of the example of colour spots? The correlated truth table does not display the negation required to render the conjunction a contradiction. There is no other way to form complex sentences in the *Tractatus*. The *Tractatus*'s answer might be that we must analyse the proposition further to eventually discover the contradiction. But the *Tractatus* nowhere says either that "a is red" is an atomic proposition or that it is not. As already discussed, this is in line with the typical logical atomist way of investigating philosophical problems, as complex propositions that have a unique logical way of formation (here, the contradiction), which is hidden (not visible in its grammar) and is very complex and which must be discovered.

This problem is still salient when Wittgenstein's returned to philosophy, as we can see how he returns to the diagrammatical display of exclusions, such as that amongst colours, in his *Nachlass*. For example, he turns back to truth tables in his essay from 1929, as we see here:

A is red	A is blue	A is red and A is blue
T	T	F
T	F	F
F	T	F
F	F	F

What is going on in 1929 is technically not significant, but it is philosophically momentous. In 1929, Wittgenstein keeps the Russellian idea of full analysis but talks about *adding rules*. Here the problem with the last truth table is not with the truth value of the last column. Note that the exclusion is not nonsensical, but it is its diagrammatic representation itself, using the *full* truth tables, which is nonsensical. The problem here is not the distinction between *sinnlos* and *Unsinn*, as Engelmann (2013) points out (p. 13). Engelmann tries to interpret the “nonsensical constructions” of *SRLF* (p. 171), conflating them to absurdities. But the exclusion of colours is not *Unsinn*. What is “nonsensical” in *SRLF* is not the exclusion itself but the truth table conjunction *authorising* the first line TT, as shown in the truth table 4 above. The exclusion of colours has, so to speak, a logical form, one which should not be eliminated from language; rather, it should be discovered in a proper logical analysis of phenomena. The problem is not, therefore, with absurdities. The problem in *SRLF* reduces to the need to discover a form of exclusion which cannot be reduced to the form of contradiction “ $p.\sim p$ ”. It is therefore about finding a new form for a *sinnlose* proposition which cannot be reduced further to a tautology or a contradiction. The use in *SRLF* of “in some sense a tautology” and “a certain kind of contradiction” corroborates this interpretation.

The problem of colours shows that the author of the *Tractatus* has to expand his logical vocabulary of *sinnlosen* expressions, based hitherto exclusively on tautology and contradiction, the old logical vocabulary being too limited. In a sense, there is a recognition that there are more *sinnlose Sätze* than tautologies and contradictions. The colour–exclusion problem breaks down, so to speak, under the identity of logical grammar and syntax exposed in 3.325, when logical syntax is considered exhausted by tautologies and contradictions. In my view, the novelty of Wittgenstein’s 1929 conception is that grammar should include more than tautologies and contradictions. *Grammatik* should also include what is conventionally called, in the philosophical

tradition, material inferences, as codified in the colour and length system. In 1929, the problem is neither with the connective “and” (WWK p. 80) nor with an exclusive disjunction, since an inclusive disjunction cannot be used either; nor is the problem just with the colour system (WWK p. 80). Nor are concepts like reddish–green or transparent white relevant in this context.

In 1929, the problem is with the scheme itself, with the free distribution of truth values. Wittgenstein realised that the combinatorial procedure in the *Tractatus* semantics had to follow additional rules. It had to be contextually sensitive. As we see in the following scheme of truth table,

A is red A is 3m long Now it’s 28°C (same for degrees of hardness, volume, sound, etc.)	A is blue A is 4m long Now it’s 29°C (same for degrees of hardness, volume, sound, etc.)
T	F
F	T
F	F

one line must be ruled out, taken away, blocked or, in more dramatic phrasing, “mutilated” in different *Satzsysteme* (Von Wright, 1996). Some combinations must be blocked *ad hoc*. To impose restriction on the formation of truth tables means to impose restrictions on other laws of propositions, as Löf would put it, such as truth functionality, extensionality and so many other typical (classical) Tractarian features. This means a dramatic philosophical turn. We must *add* rules to *restrict* logical space<sup>5</sup>.

In WWK, Wittgenstein suggests that the logical space has to be “eingeengt” (constrained, narrowed) by special rules for the inner syntax of elementary propositions which prohibit some combinations (p. 80). What is not prohibited by these rules is permitted. In other cases, truth functionality maintains its power. Intuitively, it makes no sense to speak of rules when *all* combinations and possibilities are authorised. A rule means *inter alia* to restrict, so to speak, a space of operations,

<sup>5</sup> See Silva 2016a and 2016b. For an alternative view, see Lugg 2015 and 2017.

an action field, a *Spielraum*. In *Wiener Ausgabe I*, we see a discussion about the mutilation of a line of the classical truth table for conjunction which coincides with the expression of the contrariety paradigm<sup>6</sup>. In these passages, Wittgenstein writes about a “Wegfall der ersten Linie” [“omission of the first line”] (p. 58) and also, “eine Reihe einfach durchstreichen, d. h., als unmöglich betrachten. Ich muss die ganze obere Reihe durchstreichen” [“Simply crossing out on line, i. e. consider it impossible. I must cross out the entire superior line”] (p. 58), “Die ganze ausstreichen Linie” [“Crossing out the entire line”] (p. 59) and “die obere Linie streichen” [“eliminate the superior line”] (p. 59). This discussion of the mutilation of the truth table corresponds to some discussions in a relevant way in *WWK* (pp. 64 and 92) and in *PB* § 81.

Accordingly, systematic mutilations capture some other important conceptual and logical patterns, such as:

- 1) Contrariety (since the propositions cannot be both true together, but can be false together);

p	q
T	F
F	T
F	F

- 2) Subcontrariety (since the propositions cannot be both false together, but can be true together);

p	q
T	T
T	F

<sup>6</sup> See Silva 2017 for a detailed account of Middle Wittgenstein's acknowledgment of the relevance of the distinction between contrariety and contradiction in the context of the so-called colour-exclusion problem.

F	T
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- 3) Contradiction (since both propositions can neither be both false nor true together)

p	q
T	F
F	T

#### § 4. Some problems with the principle of excluded middle

We have seen how the negation in Wittgenstein's *Satzsysteme* should express different conceptual relations in the logical pattern usually related to contraries and not to contradictions. Those two relations, of contradiction and of contrariety between pairs of meaningful propositions, are central to our discussion of the relevance of colours to the philosophy of logic and point at a transition from a logical atomist and referentialist grounded semantics to an inferentialist and holist-informed vision of the meaning of our linguistic expressions. With this distinction in mind, we can reconsider cases of inconsistency, separating them into two distinct and irreducible categories commonly raised in a confused way in the discussion of the so-called colour-exclusion problem. We can characterise the first form of exclusion, by contradiction, in this way: Two propositions contradict each other or are opposed by contradiction if and only if they can be neither false nor true together. In other words, when one proposition is false, the other must necessarily be true and vice versa. This is a reasonable explanation for the Tractarian requirement of " $p \sim p$ " always being false, i.e. having the form of a contradiction, that the truth value of " $p$ ", whatever it is, is the contradictory opposite of the truth value of " $\sim p$ ".

In a complementary way, we can characterise the second form of exclusion relevant to our discussion, namely the exclusion by contrariety, in the following way: Two propositions oppose each other by contrariety, or a pair of propositions exclude each other by contrariety, if those propositions cannot be true together. As in the case of contradiction, the fact that the propositions cannot be true together characterises the inconsistency or incompatibility within the pair. However, in the case of contrariety, the propositions in opposition can, in contrast to the contradiction pattern, be false together.

Accordingly, Brandom states, concerning the notion of incompatibility he uses to

frame his semantic inferentialism, that it...

...can be thought of as a sort of conceptual vector–product of a *negative* component and a *modal* component. It is *compossibility*. To use this semantic notion to introduce a negation operator into the object vocabulary, we must somehow isolate and express explicitly that negative component. The general semantic model we are working with represents the content expressed by a sentence by the set of sets of sentences incompatible with it. So what we are looking for is a way of computing what is incompatible with negated sentences (and, more generally, with sets of sentences containing them). Since we do not have any sort of yes/no evaluation of sentences in the picture (not even a relativized one), we cannot approach negation as a kind of reversal of semantic polarity. How else might we think about it? Incompatible sentences are Aristotelian *contraries*. (Brandom 2008, p. 126)

Instructively, paradigmatic examples for this type of inconsistency based on Aristotelian contrariety, as Brandom points out, can be found in all systems already presented in the *et ceteras* of the Tractatus 2.0131 and in the *et ceteras* of SRLF (pp. 165 and 167). In the 1929 essay, Wittgenstein seems to recognise something which he did not see in the *Tractatus*: the need to develop a phenomenology of conceptually organised domains (*Satzsysteme*). This peculiar phenomenology, as a project to be pursued at this stage of Wittgenstein’s philosophy, is strongly suggested in the last paragraph of SRLF.

Back to the first question about some lines of constructivism in Wittgenstein’s return to philosophy; some authors connect Middle Wittgenstein with verificationism, because of Brouwer’s and/or the Vienna Circle’s influence, or they even try to find, already in the *Tractatus*, some constructivist ideas, such as procedures and operations being important for sense formation. However, it is crucial to note that Middle Wittgenstein’s *Satzsysteme* are “materially thick” and “inferentially dense”; the negation of a proposition there shows the conceptual connection to many possible alternatives. As a result, in Wittgenstein’s phenomenological *Satzsysteme* (colours, temperature, volume, hardness, length, weight, height etc.), we should recognise that the principle of the excluded middle does not hold. Its validity should be restricted. The reason for this is simple: the third (or middle) term is not excluded.

Accordingly, Horn and Wansing (2015) state:

As introduced in Aristotle’s *Categories* (11b17), the genus of opposition (*apophasis*) is divided into species that include contrariety and contradiction. Contradictory opposites, whether affirmative and negative counterparts of a singular predication (*Socrates is wise/Socrates isn’t*

*wise*) or quantified expressions (*All pleasure is good/Some pleasure is not good*), are mutually exhaustive as well as mutually exclusive, while contrary opposites (*Socrates is wise/Socrates is unwise; All pleasure is good/No pleasure is good*) do not mutually exhaust their domain. Contraries cannot be simultaneously true, though they may be simultaneously false. Members of a contradictory pair cannot be true or false simultaneously; contradictories “divide the true and the false between them”. Contrary terms (*enantia*) come in two varieties (*Cat.* 11b38ff.). In immediate or logical contraries (*odd/even, sick/well*), a true middle—an entity satisfying the range of the two opposed terms but falling under neither of them—is excluded, e.g., an integer neither odd nor even. But mediate contrary pairs (*black/white, good/bad*) allow for a middle—a shirt between black and white, a man or an act neither good nor bad. Neither mediate nor immediate contraries fall under the Law of Excluded Middle [LEM] (*tertium non datur*) (my emphasis).

In other words, for any  $p$  and  $q$ , if they are opposite,  $p$  and  $q$  cannot be true together. But if the opposition is a contradictory one, either  $p$  or  $q$  must be true. However, if “a is blue”, for instance, really negates, denies or excludes “a is red”, then we must accept that both can be false together. That is, it is possible that neither of them is actually true. In this context, we have to deal with some mandatory indetermination or vagueness. For very little is known about the colour of a T-shirt or the length of a table if one states, for example, “my T-shirt is not green” or “that table is not three meters long”. If “a is red” is  $p$ , then not- $p$  need not be “a is blue”. For “a” can have all other colours, an infinite number of other colours, if we allow so. Recall that there doesn’t exist “the contrariety” of a proposition, although we have “the contradictory” of a given proposition.

By any measure, the meaning of “red” can be conveyed by pointing to a red sample but not by saying “no” or gesturing disapprovingly while pointing to a blue sample. In no dictionary is there an “x” over a sample of blue to define what red is. Here we have an important kind of asymmetry, because the affirmative proposition seems to say more than the negative, as this special “negation”, which expresses material incompatibilities in conceptual systems, “underdetermines” things; it introduces some indetermination. As we have seen, the very idea of a formal negation as contradictoriness is intended to explicate negation by understanding, symmetrically,  $\neg A$  as the contradictory of  $A$ , where the relationship of contradiction may be defined in terms of certain logical laws, such as the law of excluded middle or the law of non-contradiction. However, the negation in *Satzsysteme* cannot be only a matter of a commutator or switcher of truth conditions, because of the interesting asymmetries. As we have seen, although the conjunction of, say, “a is red” and “a is blue” may be held as a contradiction, the disjunction of both is not a tautology, regardless of an inclusive or an exclusive disjunction.



## § 5. Conclusion

The present work, inspired by Brandom's philosophy, sees a transition from a referentialist to an inferentialist account of the meaning of linguistic expressions in the context of Wittgenstein's return to philosophy. In an inferentialist approach, meaning should be given by the rules which govern some concepts and not by reference to extralinguistic objects. Since no propositions should be logically isolated in Middle Wittgenstein's *Satzsysteme*, they are actually dense in conceptual, material relations (numerous implications and exclusions); they are "inferentially thick". From this discussion, we can see an explanation of the strong verificationism characteristic of this phase in Wittgenstein's philosophy. It makes sense to defend what exists in the *Tractatus* as a kind of realistic semantics of propositional meaning, because the meaning of a proposition is unique and is exclusively determined by its truth conditions. This is captured by introducing logical operators via truth tables. However, when Wittgenstein returns to philosophy, after the full recognition of the dead end of demanding that all necessity is tautological<sup>7</sup>, some mandatory mutilations of truth tables should show inferential relations in *Satzsysteme*. That shows a reason why Middle Wittgenstein surprisingly advocates more constructive approaches to propositional sense. I have defended the notion that this is not an accident. Wittgenstein maintained, amongst other things, that to understand the meaning of propositions, one should understand their verification methods.<sup>8</sup> In other words, if  $p$  is meaningful, we should be able to verify it. From this perspective, not knowing how to verify  $p$  means not understanding the meaning of  $p$  or even doubting whether  $p$  has any meaning at all<sup>9</sup>. In realistic semantics, there are no constructive restrictions on the formation of propositions, and we can always, in principle, assign a truth value — here, true or false — to a proposition, regardless of its complexity. This Tractarian view must naturally undergo changes, as the process of formation of complex propositions cannot be made or guaranteed without determining *when and in what domains* some combinations are authorised and others prohibited.

By applying Löf's normative reading of verificationism to this discussion, we see how Wittgenstein's verificationism demands that if we have to (in ad hoc way)

<sup>7</sup> See, for example, WWK, p. 91, where Wittgenstein states that 'Tautologie ist ganz nebensächlich'.

<sup>8</sup> See PB (pp. 174, 200) and WWK (pp. 47, 53 and 79).

<sup>9</sup> I agree with Hacker (1986), as he states that: "The principle of verifications is certainly not to be found in the *Tractatus*." p. 135.

restrict the formation of molecular propositions, it is crucial to test, to verify, whether a molecular proposition is possible or not — in other words, whether a molecular proposition is allowed or not. This is done by specifying introduction rules. If a combination of elementary propositions is not authorised, say, by phenomenological rules (colour, length, volume, temperature, hardness, etc.), the assignment or determination of truth values should be restricted in each case. It is important to note that in 1929, Wittgenstein is engaged in a peculiar kind of phenomenology, where the author keeps the Tractarian idea of complete analysis but rejects the logical independence of elementary propositions (see especially the *mea culpa* in WWK, pp. 73–74). In this sense, new rules must be introduced by restricting combinations of elementary propositions; they would not be grounded in a semantics of truth conditions based on the Tractarian bipolarity.

This marks the kind of inferentialism which appears in Middle Wittgenstein's *Satzsysteme*. If we do have elementary propositions, they should all be inserted in several different systems which are logically organised through exclusions by contrariety, wherein negation codifies several, if not an infinite number of, alternatives. To operate propositions, we should know the whole system in which the propositions are inserted to check *which combinations are allowed* and *which are prohibited*. As a result, we need at least two modalities to deal with this kind of semantics, namely a combinatorial modality and a “phenomenological” or material one. The colour–exclusion problem is a good motivation for inverting the traditional priorities. First, the material one (epistemic) should be understood in terms of inferentially articulated conceptual content. Then the formal or combinatorial one can take place.

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### **Negation, material incompatibilities and inferential thickness: a Brandomian take on Middle Wittgenstein**

By 1929, after the full acknowledgment of the colour-exclusion problem, Wittgenstein had to admit that material incompatibilities presented in conceptual systems (*Satzsysteme*) could not be reduced to formal tautologies and contradictions. Wittgenstein then, in his middle period, had to examine the kind of negation which, for instance, colour systems should render, which expose not just one but many or, in some cases, infinite inferentially articulated alternatives. Here, inspired by Brandom's inferentialism (1994, 2000, 2008), I explore the idea that Wittgenstein, in his middle period, advocated a form of inferentialism based on the inferentially articulated content of propositions in *Satzsysteme*. At that time, Wittgenstein suggested that every sentence should be logically connected to many others. I call this feature inferential thickness. Therefore, I use Löf's (2013) normative read of verificationism to explain Middle Wittgenstein's holist solution to problems concerning the use of negation related to material incompatibilities and determination of propositional sense. I also investigate the distinction between contrariety and contradiction and some possible connections to a mandatory restriction of the principle of excluded middle in *Satzsysteme*.

**Keywords:** Negation · Material Incompatibilities · Inferential Thickness.

### **Negación, incompatibilidades materiales y densidad inferencial: una lectura brandomiana del Wittgenstein Mediano**

Por 1929, después de reconocer plenamente el problema de exclusión de color, Wittgenstein tuvo que admitir que incompatibilidades materiales presentadas en sistemas conceptuales (*Satzsysteme*) no se podían reducir a tautologías y contradicciones formales. Wittgenstein tuvo que examinar entonces, en su periodo mediano, el tipo de negación que, por ejemplo, sistemas de color deberían producir que no exponen sólo una, sino muchas o, en algunos casos, infinitas alternativas articuladas inferencialmente. Exploro en este trabajo, inspirado en el inferencialismo de Brandom (1994, 2000, 2008) la idea de que Wittgenstein abogó

en su período mediano por una forma de inferencialismo basada en el contenido articulado inferencialmente de proposiciones en *Satzsysteme*. En este período, Wittgenstein sugirió que cada oración debería estar lógicamente conectada con muchas otras. Yo llamo esta característica densidad inferencial. Por tanto, uso la lectura normativa del verificacionismo de Löf (2013) para explicar la solución holista de Wittgenstein Mediano a problemas referentes al uso de negación relacionados con incompatibilidades materiales y la determinación de sentido proposicional. También investigo la distinción entre contrariedad y contradicción y algunas conexiones posibles con una restricción obligatoria del principio del tercer excluido en *Satzsysteme*.

**Palabras Clave:** Negación · Incompatibilidad Material · Densidad inferencial.

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**MARCOS SILVA** is currently Associate Professor at the Federal University of Alagoas, Brazil. He has held research positions in Rio de Janeiro, Fortaleza, Leipzig and Pittsburgh and has presented his research throughout Europe. His research interests include the philosophy of logic, philosophy of language and Wittgenstein's philosophy. He is the editor of *Colours in the development of Wittgenstein's Philosophy* (Palgrave, 2017) and *How Colours Matter to Philosophy* (Springer, 2017). In 2016, he received the Fulbright Junior Faculty Member Award.

**INFORMACIÓN DE CONTACTO | CONTACT INFORMATION:** Departamento de Filosofía, Universidade Federal de Alagoas. Campus A. C. Simões, Maceió, AL, Brasil. e-mail (✉): marcosilvarj@gmail.com

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