

Logic in its Space. Wittgenstein's Philosophy of Logic in the *Tractatus*

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IN HIS *NOTES ON LOGIC*, dating from 1913, Wittgenstein betrays a moment of a categorical mood when he writes: “Philosophy consists of logic and metaphysics: logic is its basis” (NB, Appendix I, Fourth MS, p. 106). Eight years later, though, when Wittgenstein's ruminations had found their conclusive expression with the publication of the “*Logisch-philosophische Abhandlung*” – the *Tractatus* -, metaphysics saw its role somewhat curtailed to that of a side character, giving its performance in rather elusive and enigmatic aphorisms scattered through the text.

Of course, it must not be denied that metaphysical ideas still form an essential component of the *Tractatus* as whole. Nonetheless, their exposition remains ambiguous at best, and for quite systematic reasons. For according to the demarcation the *Tractatus* means to establish for “significant propositions” (*sinnvolle Sätze*), metaphysics must surely lie beyond the range of what can be meaningfully said.

In some sense, the same verdict holds also for logic whose propositions likewise are “without sense” (T 4.461) because they “say nothing” (T 6.11), given the *Tractarian* conditions for significant propositions (as laid down in what is known as the picture theory). In contrast to metaphysics, however, logic is pervasively present in the *Tractatus* and plainly out in the open, whatever the edifice in the end might be for which logic is meant to be the “basis”. Their central role, though, does not mean that Wittgenstein's remarks on logic are reliably less enigmatic or elusive than those on metaphysics. It therefore remains an assignment to see one's way to a coherent interpretation of these remarks within an overall reading of the *Tractatus*, an assignment, however, that well merits some labours. The reward is, or so I would like to contend, a powerful philosophical conception of (formal) logic that serves interests even beyond the *Tractatus'* own confinement to classical logic. The key to Wittgenstein's conception of logic is his seemingly metaphorical concept of logical space, which,

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as I will try to explain, provides not only a frame (and quite literally so) for Wittgenstein's specific approach to logic, but is also versatile enough to lend itself to logical alternatives. Wittgenstein's specific assumptions concerning logical space are closely connected with his account of sentence meaning, in particular the demand that a sentence must be definite or "determinate" in its meaning. Rival conceptions of a sentence's meaning, or rather of what it is that makes sentences meaningful, will imply diverging assumptions on the space wherein meaningful sentences are supposed to be located.

In what follows I will try to present Wittgenstein's notion of logical space as a device to unite his central views about logic as they developed in reaction to the ideas of Frege and Russell. The basic tenets concerning logic in the *Tractatus* find a conducive ambience in logical space, or so I would like to argue. The *Tractatus* offers a philosophical foundation of formal logic by explicating its emanation from what is essential to meaningful propositions. It is the logical space wherein meaningful sentences are related as meaningful items, thus exhibiting their logical relations.

§1.

Quite a number of thoughts that one encounters in the "Notebooks" (Wittgenstein 1968, pp. 89–187 for the German text, Wittgenstein 1979, for the translation to English, from here on NB) proved to be remarkably robust over the years in which the *Tractatus* took shape. The very first entry, for example, dating from August 22nd, 1914, appears unmodified as the opening for *Tractatus* (from here on: T) 5.473: "Die Logik muß für sich selber sorgen".¹

The following entry from September 2nd supplements this thought by the postulate "Wir müssen in einem gewissen Sinne uns nicht in der Logik irren können" (NB, 2.9.14),² which, in slightly modified wording, concludes T 5.473. As Wittgenstein adds, this sort of infallibility in logic is already implied by logic's "self-sufficiency", i.e. logic's taking care of itself. Further salient components of the Tractarian conception of logic, which can be traced back to entries in the *Notebooks* or the *Notes on Logic*, include the claim that tautologies "say nothing", the distinction between saying and "shewing", the idea that a significant proposition takes up a logical place, or the "fundamental thought [...] that the

¹ "Logic must take care of itself". I shall follow the practice to quote Wittgenstein in the original language, giving standard translations in the footnotes whenever the original is in German. The translation used is the first translation by F.P. Ramsey and C.K. Ogden for (Wittgenstein 1922).

² "It must in a certain sense be impossible for us to go wrong in logic".

logical constants are not proxies” (NB, 25.12.14, p. 37e). The latter statement in fact became a pivotal point for the *Tractatus* as the “Grundgedanke” in T 4.0312: “Mein Grundgedanke ist, daß die ‘logischen Konstanten’ nicht vertreten. Daß sich die *Logik* der Tatsachen nicht vertreten *läßt*”,³ which only slightly modifies the entry from Dec. 25, 1914 in the *Notebooks*. Even the statement in T 6.1261 that, in logic, process and result are equivalent, first appeared in the *Notebooks* in an entry from April 24, 1915.

While Wittgenstein’s formulations in the *Tractatus* are perhaps more lucid and clearer in their combination, a comparison with their earlier expression is still instructive. Clarity to the side, there are shifts in emphasis and there are additions, as, e.g., with the seductive phrase in T 6.13: “Die Logik ist keine Lehre, sondern ein Spiegelbild der Welt”.

The *Tractatus* is of course all but a haphazard collection of isolated aphorisms. Integrated by their order and composition, the various ideas and strains of thought that one encountered still in a somewhat fragmentary form in the *Notebooks* and the *Notes on Logic* have eventually found their systematic and coherent presentation with the *Tractatus*. This is pertinent specifically for Wittgenstein’s remarks on logic, logical form, logical constants etc., which, thus being put in perspective, find their embedding in the wider context of the *Tractarian* project. This project can be equated, if only tentatively and for present purposes, with the endeavour to state the conditions under which sentences can be seen as indeed meaningful, or, in Wittgenstein’s terms, what it is that makes propositions “significant”. This clearly involves a shift in perspective, compared to the works of Frege and Russell. Wittgenstein had no intentions to contribute to the efforts to establish a logical foundation for arithmetic, or even for all of mathematics, to which Frege and Russell were primarily devoted. Both Frege and Russell, however, already saw a need for a logical analysis of language, and Russell stated an impressive example for the strengths of such an analysis with his famous treatment of definite descriptions.⁴ Russell’s demonstration was evidence for the contribution predicate logic could make to clarify sentence meaning at the level of a sentence’s deep structure to which logical grammar is able to advance. It thus became a quite convincing idea that logical syntax, regulating conceptual relations, could be constitutive of sentence meaning, in the sense that it is able

³ “My fundamental thought is that the ‘logical constants’ do not represent. That the *logic* of the facts cannot be represented”.

⁴ The *locus classicus* being, of course, Russell’s paper (1905, “On Denoting”. *Mind*, vol. 14, num. 56: pp. 479-493). Arguably more complicated to grasp but no less impressive is Frege’s ingenious deployment of logical analysis in his treatment of arithmetical statements in his *Foundations of Arithmetic* of 1884.

to define the boundaries for the range of a sentence's potential content. This was eventually the idea that the *Tractatus* was pursuing.

However, for rather obvious reasons, Wittgenstein's logical grammar could not be presented simply as a logical system in axiomatic form (as T 6.13 obliquely points out). An axiom system already does presuppose a language in which the axioms can be stated. For Wittgenstein this must have meant that any axiom system itself will then depend on an external source of meaning and therefore cannot be the basis for sentence meaning. Although a strictly formalist stance might insist that such a formal language, or its "statements" serving as axioms, need not be interpreted, this is of no help for Wittgenstein's aim. For then there is either no meaning at all, reducing the axiom system to purely formal manipulations of typographical items, or, if it is assumed that the meaning of the axioms is given "implicitly", as Hilbert occasionally suggested, then the meaning of sentences is relative to a consistent interpretation —and any consistent interpretation will do the job. But then, if it is all about consistency and meaning is merely an "internal affair" (internal, i.e., to the system), then it does not make any sense to ask, e.g. whether a sentence, or an axiom, is true (in a substantial sense of truth). Such a consequence was clearly unacceptable for Frege as well as for Russell, given their logicist leanings, because the whole point of the logicist program was to display arithmetical statements as necessarily (analytically) true sentences, and not just as derivations from consistent axioms. It was of course unacceptable for Wittgenstein as well. The reason is that the account of meaning developed in the *Tractatus* and its quest for a logical grammar are incompatible with any conception whereby the meaning of signs or linguistic items is merely a matter of giving them a coherent interpretation. The rejection of such a crypto-formalist approach is evident from the *Tractarian* endorsement of truth, the correspondence between facts and propositions, or the projective function. In addition to the realist commitments of the *Tractarian* semantics there is also the postulate that a sentence's meaning must be definite or "determinate", and this demand is difficult to meet when meaning is allowed to be given only implicitly by the signs' relations to each other. Even before Gödel's incompleteness results opened the gates for non-standard models (of Peano Arithmetic), there were reasons to suspect that if meaning is only immanent to a system, then this cannot answer any worries concerning Wittgenstein's "significant propositions", quite besides the fact that the exposition of a formal system with its implicit definitions of terms and concepts still requires an independently given metalanguage for which questions of meaning are settled.

All this only amounts to reaffirming that the very point of departure for a logical grammar is the insight that such a grammar cannot refer to the prior and

independently given meaning of the signs if the logical syntax is to serve its purpose, namely, to yield a unique and determinate analysis of propositions wherein their meaning is revealed. In Wittgenstein's logical grammar, meaning and logical relations must be interlaced without the one taking priority over the other. The implicit idea here, that logic in a sense goes deeper than "merely" to the level of its presentation in logical systems, is evidence for the contrast between Wittgenstein's quest on the one side and the aims of Frege and Russell – the logical foundation for arithmetic - on the other side. It is also a reason why the *Tractatus* is still of interest for the philosophy of logic today: logic, over and above its role as the art and science of drawing correct inferences, is constitutive for the semantics of propositions insofar as it demarcates a space wherein meaningful sentences are located.

Wittgenstein's immediate task, however, is to specify a logical syntax, which any sign language and hence the *Tractatus*' significant propositions will obey, so that the inferential relations between propositions and their role as individually informative units (given their specific meaning) appear as two sides of one coin.⁵ If logic is to be at the heart of speaking meaningfully about the world without being able to claim priority over the possibility of meaningful sentences, rather than being embedded therein, then logic and meaning must somehow be explained as entangled or interwoven. The clever move to which Wittgenstein would finally resort to account simultaneously for meaning and logic, was to make logic vanish in the structure of meaning, having it dissolve in logical syntax.

Wittgenstein's significant propositions are those that "say something", and that makes them subject to logical constraints. For in order to convey a determinate item of information, propositions must be governed by logical syntax (T 3.325). The connection finds its explanation in a proposal made by Peter Strawson (1982). Trying to account for logical form and logical constants, Strawson suggested that to say something, i.e. to make a statement, is tantamount to the exclusion of possibilities. A meaningful proposition p , qua being meaningful, is incompatible with at least some possible states of affairs. To exclude possibilities as incompatible with p , however, includes a tacit reference to the totality of possible states of affairs. Deploying the spatial metaphor, Wittgenstein can therefore say in T 3.42 "[...] Der Satz durchgreift den ganzen logischen Raum",⁶ and in this sense logic is adequately conceived as the structural underpinnings of significant propositions. These underpinnings, however,

⁵ This inter-relatedness between sentence meaning and logic and the observation that an account of the one cannot be severed from the other in The *Tractatus* is emphasized in Gustafsson (2014).

⁶ "The proposition reaches through the whole logical space".

cannot be stated adequately and in a non-circular way in axiomatic form, as I have tried to argue. Instead, the solution is to identify logical relations via the combined effects of the exclusions by individual propositions, i.e. by the possibilities that they exclude in complementary and coordinated ways.

§2.

The *Tractatus* offers some characterization of logic and its laws as it is indispensable if the aim of a logical grammar is to deserve any credit. Wittgenstein's alternative to a logical system, textbook style, are remarks that jointly circumscribe how the workings of logic, ultimately for the purpose of a logical grammar, are to be understood. These remarks are anchored with some leeway in five salient claims that seem to me to capture the quintessence of the *Tractarian* conception of logic.

(1) "Die Logik muß für sich selber sorgen" (T 5.473), which is to say that there is nothing more fundamental than logic itself, and that there is no logic (and no meaningful language) behind logic, a point on which Wittgenstein insists against Russell's theory of types, for example in T 6.123 "Es ist klar: Die logischen Gesetze dürfen nicht selbst wieder logischen Gesetzen unterstehen".⁷

(2) The *Grundgedanke*: "Mein Grundgedanke ist, daß die »logischen Konstanten« nicht vertreten. Daß sich die *Logik* der Tatsachen nicht vertreten läßt" (T 4.031). The idea that logical constants do not stand for objects in a sentence and in this sense do not represent anything is as such hardly spectacular. There are many expressions in any language that do not name anything. Nonetheless, the *Grundgedanke* is essential for Wittgenstein's view of logic because it supports his central claim that logic must be *immanent* to language in the sense already indicated. In T 6.13 Wittgenstein states, obviously addressing Frege, "Die Logik ist keine Lehre, sondern ein Spiegelbild der Welt"⁸, thereby denying that logic has (or is) a proper subject matter. Nothing of substance can be said about logic (logical objects, logical constants, logical functions etc.), at least not by way of significant propositions according to *Tractarian* standards. Logical signs, including the usual logical constants, do not denote or refer to anything: "Hier zeigt sich, daß es »logische Gegenstände«, »logische Konstante« (im Sinne Freges und Russells) nicht gibt" (T 5.4).⁹ They are, at best, punctuation

⁷ "It is clear that the laws of logic cannot themselves obey further logical laws". See also T 3.331.

⁸ "Logic is not a theory but a reflection of the world", - a slightly problematic translation, because the German "Lehre" might be more aptly rendered by "doctrine".

⁹ "Here it becomes clear that there are no such things as 'logical objects' or 'logical constants' (in the sense

marks (T 5.4611). A corrected perspective and something like a conclusion from the *Grundgedanke* and some further remarks on logical constants and logical objects is offered in T 5.47:

“ [...] Man könnte sagen: Die Eine logische Konstante ist das, was alle Sätze, ihrer Natur nach, mit einander gemein haben. Das aber ist allgemeine Satzform”.¹⁰

The *Grundgedanke*, finally, has ramifications that connect it closely to two further components of the *Tractarian* conception of logic. These are, first, the claim that tautologies say nothing, and second, the fundamental distinction between saying and showing.

(3) That the sentences of logic say nothing is repeatedly affirmed in the *Tractatus*, for example in T 6.11: “Die Sätze der Logik sagen also Nichts. (Sie sind die analytischen Sätze)”. Earlier, in T 4.461, we were informed that tautologies (as well as contradictions) show that they say nothing and that they are therefore meaningless (they are “without sense”). T 5.142 and also T 5.43 likewise emphasize that tautologies say nothing. It is of course a hallmark of the whole *Tractarian* conception of language that under its account of logical propositions these must turn out to be meaningless because they cannot purport any state of affairs as being the case – they do not discriminate between possibilities. However, being without sense is not the same as being nonsense. Tautologies arise from suitable combinations of significant propositions, and it therefore is convenient to see them as limiting cases of propositions. On any reading, however, tautologies play an important role for the *Tractarian* endeavour: without saying anything they still show something. As T 6.124 somewhat misleadingly starts “Die logischen Sätze beschreiben das Gerüst der Welt”, only to add for clarification: “oder vielmehr, sie stellen es dar”,¹¹ thus pointing towards the distinction between saying and showing, which was introduced in remarks following T 4. The upshot is that while the sentences of logic are properly speaking without semantic content because they do not convey any information on how things are supposed to be, they still have a semantic function insofar as they reveal the range within which significant propositions are situated. And they can do so, precisely because they are the limiting cases that, while being syntactically correct, do no longer what significant propositions are supposed to do: to say how things are by

of Frege and Russell)”.¹⁰

¹⁰ “[...] One could say: the one logical constant is that which *all* propositions, according to their nature, have in common with one another”.

¹¹ “The logical propositions describe the scaffolding of the world, or rather they present it”.

excluding options that are incompatible with their content.

(4) It is perhaps of interest to note that the *Grundgedanke* (T 4.0312) is introduced in the context of Wittgenstein's crucial distinction between saying and showing. In T 4.022 we learn "Der Satz *zeigt* seinen Sinn. Der Satz *zeigt*, wie es sich verhält, *wenn* er wahr ist. Und er *sagt*, *daß* es sich so verhält".¹² Just a few remarks later, in T 4.121, this distinction between saying and showing is connected with logical form: "Der Satz *zeigt* die logische Form der Wirklichkeit", which finds further elucidation in T 4.1212: "Was gezeigt werden *kann*, *kann* nicht gesagt werden".¹³

There are several issues concurring in these remarks and the saying-showing distinction in general. Allusions to a truth-conditional account of sentence meaning to the side ("Der Satz *zeigt*, wie es sich verhält, *wenn* er wahr ist"), it is in particular a bisection into two levels, or rather spheres of meaning that is brought about by this distinction. Thus, logical sentences are not sheer nonsense, incomprehensible balderdash, mere scribblings or noises, because, although being border cases of propositions, they do "show" something. *What* it is that they show is perhaps a bit blurred by the suggestion that it is "the scaffolding of the world", while one would rather expect something like the structural frame for meaningful sentences. However, we are told that this does not make much of difference in T 6.12: "Daß die Sätze der Logik Tautologien sind, das *zeigt* die formalen – logischen – Eigenschaften der Sprache, der Welt"¹⁴, the equation between language and world being justified by the isomorphism that comes with the picture theory. Now, whatever it is that a sentence shows, it cannot be outside of or independent from logical relations, and a tentative assumption could be to say that a proposition's logical form is enunciative for the logical space wherein the proposition (as a proper, meaningful proposition) is located. Under this suggestion, the distinction between saying and showing adds momentum to the contention that logical relations are internal to language in the sense that logic is a "reflexion" of the world, not a doctrine to be exposed separately from an account of significant propositions.

(5) A further tenet that can be identified by reverse engineering in order to

¹² "The proposition *shows* its sense. The proposition *shows* how things stand, *if* it is true. And it *says*, that they do so stand".

¹³ T 4.121 The propositions show the logical form of reality. They exhibit it.

T 4.1212 What *can* be shown cannot be *said*.

¹⁴ "The fact that the propositions of logic are tautologies *shows* the formal – logical – properties of language, of the world".

delineate the *Tractarian* conception of logic, and the last one for our concerns here, is the claim that, in logic, process and result are equivalent (T 6.1261); hence, no surprises in logic (T 6.1251; 6.1261). This contention resonates neatly with our earlier pillars for the *Tractarian* view of logic. Equating process and result in logic implies a “static” conception of logic, in contrast to accounts that see logic rather as an instrument of information-processing for the human mind with its limited capabilities. Remarks following T 6.126 further emphasize Wittgenstein’s non-epistemic conception of logic, as, for example, in T 6.1265: “Immer kann man die Logik so auffassen, daß jeder Satz sein eigener Beweis ist”.¹⁵

Such a position is a static one, in contrast to a more “dynamic” view, because logic is not seen as a means or an instrument to move safely between information states on the basis of specific items of information. Rather, logic is, according to Wittgenstein’s understanding, which he shares with Frege and Russell, what delimits the realm of possible states of affairs, as a remark early in the *Tractatus* endorses: “Die Logik handelt von jeder Möglichkeit, und alle Möglichkeiten sind ihre Tatsachen” (T 2.0121).¹⁶ Various catchy formulations contribute to a picture of logic as the structural constraint on what possibly can be the case: logic as the “scaffolding of the world”, the limits of logic as the limits of the world (T 5.61), the assertion that a god’s creation of a world wherein certain propositions are true is already the creation of a world where all of their consequences are true (T 5.123), and many more. In all these remarks, logic is never on the side of an epistemic subject and always on the side of the facts, or rather *Sachverhalte*. For such a realist, states-of-affairs related conception, logical techniques as reasoning devices must appear as quite unnecessary, as indeed Wittgenstein confirms in T 5.132: “»Schlussgesetze«, welche – wie bei Frege und Russell – die Schlüsse rechtfertigen sollen, sind sinnlos, und wären überflüssig”.¹⁷

The informational discriminations as the roots of all logic, effected by propositions, reside firmly on the side of the states of affairs, never venturing out to epistemic subjects and their speculations about what is the case (under such and such a hypothesis). Given such a perspective, process and result must be equivalent.

Let us briefly take stock of what we have garnered so far.

¹⁵ “Logic can always be conceived to be such that every proposition is its own proof”.

¹⁶ “Logic treats of every possibility, and all possibilities are its facts”.

¹⁷ “Laws of inference, which – as in Frege and Russell – are to justify the conclusions are senseless and would be superfluous”.

Logic marks the bounds of possibilities. Possible states of affairs are the content of meaningful sentences. The limits of what is possibly the case are not themselves a state of affairs, properly speaking, because they are a structural constraint imposed on possibilities and co-possibilities. Therefore, logical propositions cannot “say” anything, where saying something means saying “how things stand”. Not being ordinary propositions, logical sentences cannot be sentences “about” anything, and even less so, because logical constants do not name anything (the *Grundgedanke*). As any theory or doctrine would have to consist in meaningful sentences, it is a further consequence that logic cannot be a doctrine and that there cannot be a logical theory: logic must take care of itself. And it does so by demonstrating its relational requirements in the interactions between meaningful propositions, or more precisely, how they may combine to meaningful or no longer meaningful propositions. We are thus left with a conception of logic as the formal structure of what is conceivable or possible, i.e. what can be expressed by significant propositions. The core idea behind that is that a proposition is meaningful only to the extent that it is able to distinguish between possible states of affairs, discriminating between those that are, and those that are not compatible with its content.

Let us see whether Wittgenstein’s metaphor of a logical space can now further our understanding of this idea.

§3.

Logical space enters the stage in the first scene in T 1.13: “Die Tatsachen im logischen Raum sind die Welt”¹⁸, and incidentally this is also the first mention of anything related to logic. It is a remark, or so I am inclined to read it, that already indicates the “structural account” of logic that is to follow. Already the *Notebooks* have attested the centrality of the notion of logical space for the development of Wittgenstein’s conception of logic, albeit pointwise, in the guise of an element of logical space as “logical place”. Though convinced that a logical place is “the possibility of an existence”, very much like a place in physical or geometrical space (NB, 7.11.14), Wittgenstein still grapples with the question what the “logical place” really is. The answer can finally be found in the entry in the *Notebooks* from November 19th, 1914, and it became, only slightly modified, T 3.41: “Das Satzzeichen und die logischen Koordinaten: das ist der logische Ort”¹⁹, which is an elaboration on remark T 3.4 whose first sentence is: “Der Satz

¹⁸ “The facts in logical space are the world”.

¹⁹ “The propositional sign and the logical co-ordinates: that is the logical place”.

bestimmt einen Ort im logischen Raum”.²⁰ In fact this is Wittgenstein’s central claim regarding logical space and its role in relation to propositions and it is the clue to understand how the notion of logical space manages to integrate logic and language in the *Tractatus*. A statement specifies a place in logical space by isolating or singling out possibilities as actualized, given the statement’s truth. Propositions, if meaningful, thus make distinctions among all possible states of affairs insofar as their individual truth is not compatible with *all* possibilities, because, as Peter Strawson once put it: “[...] if it excludes nothing, it says nothing” (Strawson 1976, p. 4). They localize a place in logical space, i.e. they single out that place as a sphere of realized possibilities by giving us, in typographical appearance as “propositional signs”, the “logical coordinates” of that sphere out of all possible states of affairs. The orientation offered by logical coordinates in terms of an “area” or “sphere, wherein we find ourselves, if the proposition is true, invites the topological notion of neighbourhoods, which can be conceived in this connection as (open) sets of possible states of affairs. The difference between geometrical coordinates, that localize (sets of) points in an n-dimensional space \mathbb{R}^n , and logical coordinates results from demands on the structure of the underlying space, and this is induced by the nature of a meaningful sentence in the *Tractarian* sense. Something along these lines seems also to motivate Strawson’s attempt to extract the notions of logical form and logical constants from a proposition’s essential feature, namely, to *exclude* possible states of affairs: “What is distinctive about forms and constants of logic is that their whole force or meaning can be explained without drawing on any materials other than those which we are given with the notion of a proposition” (Strawson 1976, p. 4).²¹

Returning to the notion of logical space, as it is determined by a proposition, we repeat that with a proposition and the logical place it specifies the whole logical space must be given. This is more than just a bare presupposition. The ways in which a sentence is related to other sentences accounts for the structure of the logical space, as we are told in T 3.42:

Obwohl der Satz nur einen Ort des logischen Raums bestimmen darf, so muß doch durch ihn schon der ganze logische Raum gegeben sein.

(Sonst würden durch Verneinung, die logische Summe, das logische Produkt, etc. immer

²⁰ “The proposition determines a place in logical space”.

²¹ The view that logic and logical relations are “innate” to the concept of a meaningful proposition is also expressed by Juliet Floyd in (Floyd 2005, p. 88). For both Floyd and Strawson it is a statement’s ability to be true, and hence to be false, on which the entire logical apparatus rests.

neue Elemente – in Koordination – eingeführt).²²

It is this remark that strongly encourages a topological interpretation, which in turn may give weight to Wittgenstein’s notion of logical space, lifting it above the merely metaphorical. The key here is the postulate in the parentheses that for any two “points” in logical space (ignoring for the moment negation or “denial”) there exists also their logical sum and their logical product. Formally, this is equivalent with the assumption that the points in logical space form a lattice. Given a set S and a partial order \leq on S , i.e. a reflexive, transitive and anti-symmetric relation on S ,²³ a lattice is a structure (S, \leq) such that any two elements x, y in S have a greatest lower bound (meet) $x \bar{\wedge} y$ as well as a least upper bound (join) $x \vee y$. Meet and join are the logical product and the logical sum, respectively, and the meet $x \bar{\wedge} y$, for any x, y in a lattice (S, \leq) , is defined by

$$\text{i. } x \bar{\wedge} y \leq x; x \bar{\wedge} y \leq y$$

and

$$\text{ii. for any } z \text{ such that } z \leq x \text{ and } z \leq y \text{ it is also the case that } z \leq x \bar{\wedge} y$$

(with dual conditions for the join). Under a logical reading of the meet $x \bar{\wedge} y$ as conjunction of x and y the relation \leq has also a natural logical interpretation, as one can see by observing that the definition: $x \leq y$ can simply be read as “ x implies y ”.

The *Tractarian* conception of logic is faithful to classical logic throughout, following herein the lead of Frege and Russell, and the propositional, “non-quantified” part received an elegant exposition of its truth-functionality in the remarks around T 5.1 and T 6.1.²⁴ Now, classical logic requires more algebraic structure than arbitrary lattices will provide. For example, conjunction is distributive over disjunction, which is mirrored in *distributive* lattices whose joins

²² “Although a proposition may only determine a place in logical space, the whole logical space must already be given by it. (Otherwise, denial, the logical sum, the logical product, etc., would always introduce new elements – in co-ordination)”.

²³ A relation R is antisymmetric iff for a u, v in its field uRv and vRu implies $u = v$.

²⁴ Alternative, non-classical logic systems had to await the 1930s for their formal development.

and meets satisfy

$$(*) \quad x \bar{\wedge} (y \vee z) = (x \bar{\wedge} y) \vee (x \bar{\wedge} z)$$

for every x, y, z .

These algebraic considerations would not be of interest here, were it not for the presumption that with Wittgenstein's remark, that any proposition gives us the whole logical space, we are just one step apart from an interpretation of the logical space as a topological space. The reasoning is as follows: first, that individual propositions, while only determining a logical place, give us the whole logical space is explained by reference to meets and joins (logical products, logical sums) for which, second, we must assume distributivity for logic's sake. Third, although there are several remarks where Wittgenstein writes of tautologies, in particular in remarks following 6.1, there can be, *sub specie* of logical grammar, strictly speaking and in a generic sense only one tautology and one contradiction. In a logically trivial and philosophically unassuming sense all tautologies are equivalent, simply because they all say the same, namely nothing.

However, with just one tautology, and its dual, the one contradiction, we have two designated objects 0, 1 such that 1 is implied by every proposition, while 0 implies all propositions. A lattice (S, \leq) with a top and a bottom element, 1 and 0 respectively, is a *complete* lattice in the sense that every subset of S has a join and a meet. Fourth, a distributive lattice that is complete is also called a *frame*, and with a frame structure one finally gets close to topological spaces. Let S be a set and let P be a family, i.e. an indexed set of subsets of S (i.e. P is a subset of the powerset of S), such that,

- i. if $Q \subseteq P$ then $\bigcup Q \in P$
- ii. if $Q \subseteq P$, for Q finite, then $\bigcap Q \in P$

so that P consist of subsets of S and is closed under arbitrary unions and finite intersections, then P is a topology on S and (S, P) is a topological space.

The similarities to Wittgenstein's logical space now come into view. Take "places" in the logical space as sets of points in the logical space. Because conjunctions or disjunctions of meaningful propositions will also determine places in the logical space, the set of places must be closed under (finite) intersections and unions, i.e. under meets and joins, just as it is required for a

topological space. A topology generally postulates the existence of arbitrary unions, but infinite disjunctions will not be allowed by the logical syntax. However, as there is a top element 1 (the tautology), the topology will not be affected by such a grammatical restriction.

Yet, assuming that all of this makes sense, what do we gain for an understanding of Wittgenstein's philosophy of logic? A topological look at logic is less outlandish than one might assume, although it has been exerted mainly in connection with constructivist versions of logic, e.g. intuitionistic logic.²⁵ In an abstract way, the topological perspective combines and generalizes a geometric intuition, exploited by Wittgenstein in his metaphor of the logical space, with the formal rigor of algebra. The foundational aspect of Wittgenstein's logical space consists in its capacity to allow discriminations in a logically relevant way: meaningful sentences exclude possibilities by specifying their "co-possibility sphere" in logical space. The adequate formal structure for this finds its expedient presentation in a topological setting, where Wittgenstein's logical places are the open sets of a topology. However, Wittgenstein must demand a rather specific structure for his logical space to ensure that it accommodates classical logic. Differences between classical logic and weaker, or more constructively minded systems of logic (e.g. intuitionistic logic, minimal logic, or even relevance logic) are typically related to diverging interpretations of negation and find their expression in modified rules for negation. How can this be captured by operations on logical places? Let p be a proposition that determines a logical place. The operation that gives us the logical place for $\neg p$ must be such that the double negation $\neg\neg p$ takes us back to p , as is evident from, for example, T 4.0621, where it is explicitly asserted that a sentence says the same as its double negation. In topological terms, the logical place determined by p will be an open set and p 's negation will then simply correspond to the (relative) complement of the open set corresponding to p . The complements of open sets in a topology, however, are closed sets, where the contrast between open and closed sets can be explained for the moment by an example from the real line: the interval $(0, 1)$ is an open set, consisting of all x such that $0 < x < 1$, while $[0, 1]$ is the closed set $\{x : 0 \leq x \leq 1\}$. The intuition behind closed sets is, as the example highlights, indeed one of "determinateness"; a set is closed if it contains its "boundary". Wittgenstein insists, as we noted above, on the determinateness of sense, and T 3.23 explicitly postulates that a proposition's meaning must be definite, claiming the "Forderung der Bestimmtheit des Sinnes". The determinateness of meaning is in the Tractarian setting a necessary condition to identify the meaning of a sentence

²⁵ See, for example, Sambin (1989).

with its truth conditions, these being an “expression of the agreement and disagreement with the truth-possibilities of the elementary propositions” (T 4.431). And this sort of agreement (or disagreement) must be definite. Now, if the meaning of a sentence, as the logical place it determines, were in some sense indeterminate, how could we be sure that double negation takes us back to where we started? In T 4.431 Wittgenstein seems to argue against Frege that the determinateness of $\neg p$ requires the determinateness of p , in the sense that its agreement or disagreement with truth-possibilities is sufficiently definite. If sentence meaning would instead consist in, say, verifiability conditions then the injunction for determinateness would indeed become doubtful. We see that the lemon is yellow, we do not see that it is not blue, as Andrzej Grzegorzcyk once remarked in connection with his epistemic motivation for intuitionistic logic (Grzegorzcyk 1964, p. 596). Negation, accordingly, is governed by markedly different rules in intuitionistic logic, whose semantics can be stated in verifiability conditions, or is absent altogether, as in Abramsky’s “logic of finite observations”, where it is accepted that (finite) observations are inconclusive to decide on empirical claims.²⁶

The intuition underlying open sets translates into algebraic properties that indeed qualify topology for metalogical investigations. Via the link of lattice structures, topological spaces are closely connected with Heyting algebras, which differ from Boolean algebras in their complement operation (the algebraic counterpart to negation).²⁷ The technical details, however, are unimportant for the present context. Our focus was on Wittgenstein’s notion of logical space and the extent to which it can serve foundational purposes for logic. What hopefully has transpired is that Wittgenstein’s notion of logical space is more than just a metaphor. Combined with conditions on meaningful propositions, where these are supposed to determine places in logical space, the logical space provides a sufficiently rich structure for harbouring logical relations. Wittgenstein’s insistence on a determinate meaning, and in particular his commitment to classical negation, is reflected in the formal, algebraic properties of his logical space. If one’s philosophical predilections mandate certain reservations concerning realist conceptions of semantics and logic, and semantic realism comes with a metaphysical price tag, then the concept of a logical space is still versatile enough to allow for suitably modified logical structures. The *Tractatus*

²⁶ See Vickers (1989, pp. 5-11), for a brief exposition of Abramsky’s logic of finite observation and its relation to topological ideas.

²⁷ For Heyting algebras, it is convenient to introduce an operator \Rightarrow , subject to $x \bar{\wedge} y \leq z$ iff $x \leq y \Rightarrow z$, so that logical negation $\neg x$ can be defined by $x \Rightarrow 0$ as a pseudo-complement, see Ono (2019, pp. 95-96).

offers a philosophically coherent foundation for classical logic by which Wittgenstein tried to overcome deficiencies he suspected in the works of Frege and Russell. It is, however, also of importance in its own right. Wittgenstein's foundational efforts rest on his analysis of propositions and propositional content. The logical or inferential relations between meaningful propositions determine the shape of Wittgenstein's logical space. If they are to convey information, then propositions must select a place in logical space; only as tautologies leave sentences the whole logical space and are compatible with any state of affairs. All this, however, assumes a very specific conception of what it is to be, as a proposition, a meaningful item of information. Accordingly, alternative conceptions of sentences as informational items will be tied to different formulations of their inferential relations. In this sense, Wittgenstein's logical space is an open invitation to philosophical reflections on logic beyond the *Tractarian* commitments to classical logic, and still one from which much can be learned.

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Logic in its space. Wittgenstein's philosophy of logic in the *Tractatus*

The paramount role of logic in Wittgenstein's *Tractatus* is undeniable and must be obvious to anyone even on a cursory reading. Yet, Wittgenstein's formulations often appear metaphorical when he sketches his ideas on logic and its relation to sentence meaning. Sometimes, they seem more apt to invite loose philosophical associations than pinning down rigorously technical details. This impression notwithstanding, the *Tractatus* still offers one of the deepest philosophical accounts of modern logic and it does so precisely through its suggestive exposition. It is, in particular, the spatial analogies, Wittgenstein's 'logical place' and 'logical space', which are crucial in this connection, and while they carry a structure that is specific for Wittgenstein's own conception of logic, they are sufficiently general to accommodate also alternative conceptions of logic. Thus, even 100 years after its first publication, Wittgenstein's *Tractatus* remains a source of inspiration and a highly valuable one for the philosophy of logic.

Keywords: Logical Operations • Logical Space • Topology • Sentence Meaning.

La lógica en su espacio. La filosofía de la lógica de Wittgenstein en el *Tractatus*

El papel fundamental de la lógica en el *Tractatus* es innegable y debe ser obvio a cualquiera aun en una lectura superficial. Sin embargo, las formulaciones de Wittgenstein muchas veces parecen ser metafóricas cuando él esboza sus ideas sobre la lógica y su relación con el significado de las oraciones. A veces dan la impresión de ser capaces de invitar asociaciones filosóficas libres más bien que dejar fijados detalles técnicos de manera rigurosa. Esta impresión no obstante, el *Tractatus* ofrece aún así una de las explicaciones filosóficas más profundas de la lógica moderna y lo hace, precisamente, por medio de su exposición sugestiva. Son, en particular, las analogías espaciales, el «lugar lógico» y «espacio lógico» que son cruciales en este contexto, y mientras portan una estructura que es específica para la concepción de la lógica propia de Wittgenstein son suficientemente generales para dar lugar también a concepciones alternativas de la lógica. De esta manera, aun 100 años después de su primera publicación, el *Tractatus* de Wittgenstein sigue siendo una fuente de inspiración, de hecho, una fuente altamente valiosa para la filosofía de la lógica.

Palabras Clave: Operaciones lógicas • Espacio lógico • Topología • Significado de las oraciones.

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