Reconciling Anti–Nominalism and Anti–Platonism in Philosophy of Mathematics

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IN APRIL, 2016, ACHILLE VARZI AND MARCO PANZA hosted a conference at Columbia University on Reconciling Platonism and Nominalism in Philosophy of Mathematics. The remarks to follow are a lightly edited version of my presentation at the conference. I should begin by explaining, however, that despite my appearing on the program of a conference with the title indicated, I do not believe that platonism properly so called can be reconciled with nominalism in the philosophy of mathematics. But then *platonism* is often used improperly, in an historically absurd sense, as a mere synonym for anti–nominalism. And I do believe that opposition to nominalism can be reconciled with opposition to platonism in any historically serious sense of *platonism*. Indeed, there are many ways to combine anti–nominalism with anti–platonism, of which I have long advocated a particular one.

I will begin here by restating (though not rearguing) the main features of my preferred combination, then turn to contrasting and critical views, motivating additions and amendments. But let me first acknowledge, here at the outset, that what I will be sketching is a view that is mine in the sense that I subscribe to it, not in the sense that I would claim to have originated of it, though the history of the view is one topic I will not go into even in the most summary fashion, since that would inevitably soon land us in thickets of exegetical controversy. Suffice it to say that influences from Carnap (1950) will be evident, for instance, though there are also differences.

§ 1. A Kind of Anti–Platonism

To begin with platonism and opposition thereto, properly speaking the platonist label should in my opinion be applied only to views that longstanding tradition or current scholarship ascribes to Plato or his avowed followers and admirers in

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ancient and modern times. I see three such views in philosophy of mathematics. One platonistic claim is that mathematical objects are more real than physical objects. This view is, of the three I'll be considering, the one I would have the fewest qualms about attributing to the historical Plato, but it is also a view that seems to have no present–day adherents —"as real as" seems to be as high as anyone is willing to bid today— and so it need not concern us further.

Another platonistic claim is that just as we have knowledge of physical objects through vision, so we have knowledge of mathematical objects through an analogous faculty. A view of this kind, though about forms rather than mathematicals, was rightly or wrongly associated with Plato in antiquity, notably in an apocryphal tale of a visit by Diogenes the Cynic to the Academy. There he found Plato lecturing on his forms, and using the nouns *tableness* and *cupness*, and complained that he could see the table and the cup, while tableness and cupness he could in nowise perceive. Plato replied that this was because he had eyes with which to perceive tables and cups, but not intellect, by which tableness and cupness are seen. This story balances another in which the Cynic gets the better of the broad–browed sage: Diogenes, having heard of Plato's definition of *man* (that is, *anthropos*, human being) as a featherless biped, comes to the Academy and holds up a plucked chicken, saying "Behold Plato's man!" (See Hicks 1925, Book VI, for a series of such fables.) Among recent figures we find in the works of Gödel (1947, especially) some passages on mathematical "intuition" that have been read, rightly or wrongly, as suggesting a view similar to that attributed to Plato. I don't want get into exegetical rights and wrongs about Gödel any more than about Plato, and will just note that since Gödel's death it has been hard to find a prominent philosopher of mathematics who even appears to hold a similar doctrine, and so this view perhaps need not much concern us, either.

Yet another platonistic claim is that the demiurge created the world according to a mathematical plan, and that when we discern in the world a mathematical pattern, we are to that extent recapturing its creator's thoughts. This view is, of the three I am considering, the one least specific to Plato, and it perhaps really came into its own less in antiquity than with the Italian Renaissance and especially Galileo —whose views, however, Professor Panza has characterized in oral comments as not exclusively Platonic, but compatible with Christianized Aristotelianism. In a famous passage from *The Assayer* (available in English in Drake 1957, pp. 237–238, and ubiquitously quoted elsewhere), the universe is called a grand book that cannot be read unless one first learns the language in which it is written, which is that of mathematics.

William James (1907, p. 9), early on in his account of what he calls "pragmatism", ascribes such an attitude to the whole sweep of ancient and early

modern science in a memorable passage that manages to work in unmistakable allusions to Aristotle, Euclid, Kepler, Descartes, and Linnaeus, as well as Galileo, in the space of a few sentences. James says that when the first laws were discovered, thinkers were so carried away that they imagined they had read God's mind: "He made velocity increase proportionally to the time in falling bodies [for one example] … and when we rediscover any one of these his wondrous institutions, we seize his mind in its very literal intention." Naturally, as we pass from the theistic and deistic atmosphere of seventeenth and eighteenth century science to the nineteenth and twentieth centuries' more agnostic or atheistic climate, the theological language must become less acceptable to many, at any rate if taken literally rather than metaphorically. Nonetheless, something of the kind of "platonistic" attitude exemplified by Galileo remained for James influential enough that he defined his own position by opposition to it.

For in the material I have just quoted from him he was describing rather than endorsing an attitude, and his very next sentence begins with the word *but* and begins to express his rival picture. The kind of anti–platonist view to which I myself subscribe is opposed to the platonistic picture much as James's view was. My kind of anti–platonism, as it applies to the existence of mathematical objects, may be described as follows. Suppose there were extraterrestrial intelligences who had a science that met their practical needs about as well as ours meets ours, but which did not have a mathematical component of the same shape as ours, by lacking any apparatus of distinctively mathematical objects. If, as per platonism, our scientific picture of the world succeeds in providing a God's–eye view, literally or metaphorically speaking, and the aliens' differs from ours, then the aliens' scientific picture of the world fails to provide a God's–eye view, and so they are getting wrong something that we are getting right. The anti–platonism I endorse maintains that, on the contrary, *no clear sense has been made out in which hypothetical extraterrestrials would be doing wrong, by excluding or ignoring mathematical objects, something that we are doing right, by including or recognizing them.*

For there to be any substance to this form of anti–platonism, it must be at least conceivable that there could be extraterrestrials of the kind imagined, aliens with a developed science involving no distinctively mathematical objects. But I think that nominalistically–inclined writers of the last decades of the twentieth century (notably Chihara 1973 and 1990, Field 1980, Hellman 1989) have given us enough of a sketch of what such an alien science might be like to make this assumption plausible. In embracing an anti–platonism of the kind I have just sketchily described, I am endorsing the work of the nominalists of decades past as a contribution to theoretical exobiology, helping us to imagine what the

intellects of intelligent aliens might be like in certain respects.

§ 2. A Kind of Anti–Nominalism

For there to be any substance to the form of exobiological anti–platonism in question, it must equally be the case that our science does involve distinctively mathematical objects. I will for the bulk of this discussion simply take this for granted, considering qualifications only towards the end of my remarks. In so doing, I am taking the usual mathematical formulations of physics, for instance, at face value, and therewith reject the work of the nominalists of a couple of decades ago as any contribution to psychology or linguistics. That is to say, I am rejecting any hermeneutic claims to the effect that nominalistic reconstruals reveal what, deep down, and despite misleading superficial appearances, our scientific theories have really meant all along.

I equally reject the work of the nominalists in question as contributions to mathematics or physics, rejecting any revolutionary claims to the effect that nominalistic reconstructions of various theories are better science. This is actually a double rejection: on the one hand, of any claim that nominalistic reconstructions are better than orthodox theories by our scientific standards; and on the other hand, of any first philosophy that would claim to provide higher–than–scientific standards by which our scientific standards might be judged. My anti–nominalism consists precisely in rejecting the work of nominalists of decades past as contributions to *anything more than* theoretical exobiology.

The net result is that I would be prepared to repeat in the philosophy seminar room the kinds of things that are said outside it by mathematicians and scientists, and by myself when speaking on mathematical or scientific subjects. And I would further be prepared to endorse what are obvious consequences of a face–value reading of things said in mathematical and scientific contexts. For instance, if outside the philosophy seminar room I were expounding Gödel's work in mathematical physics, I would assert that he proved the existence of solutions to the field equations of general relativity admitting closed time–like curves. Inside the philosophy seminar room I would be willing to repeat this assertion, not claiming that when I first uttered it I had my fingers crossed, or was only speaking figuratively, or was only offering a useful fiction. And when I assert that there exist solutions to the field equations of general relativity admitting closed time–like curves I will, making explicit what is implicit in a face–value reading of such an assertion, add that since solutions to differential equations are functions of a certain kind, there exist such things as functions.

Yet further, in the mathematical physics seminar room, functions, sets,

numbers, and the like are never ascribed coordinates in space, or dates in time, or mass, or charge. When the missing mass problem in cosmology is discussed, for instance, it is never suggested that dark matter may be composed of numbers rather than neutrinos. In the philosophy seminar room I will accordingly assert that sentences attributing coordinates, dates, masses, charges, and so on to them have no use, and hence no meaning, in our scientific language. I will sometimes engage also in the practice of speaking in the material rather than the formal mode, and say that numbers, sets, functions, and so on have no coordinates in space, have no dates in time, have no mass, have no charge, and so on; or that numbers, sets, functions, and so on are "not the sorts of things that can have" coordinates or dates or mass or charge.

I may even sometimes let myself go and say that however numbers are, they never were nor will be nor could have been otherwise. But this manner of speaking presents certain dangers of being misunderstood, and I will retreat to the formal mode if misunderstanding threatens. And I definitely will not say that numbers are "eternal" and "necessary", since that manner of speaking positively *invites* misunderstanding, by giving far too positive and far too metaphysical a sound to what is essentially a negative grammatical point, that the usual inflections for tense and mood that apply significantly to most sentences have no serious application to purely mathematical sentences: There are infinitely many prime numbers, but to ask how long there have been or what if there hadn't been such numbers is to commit a kind of grammatical solecism. And it is not that our grammar excludes such usages in order to the reflect the fact that numbers exist and are related to each other as they are eternally and necessarily, in a way immune to change and contingency, but rather the grammatical fact is the primary one, and talk of immunity of to change and contingency, if we are incautious enough to allow ourselves to be carried away and indulge in it, is merely a kind of projection of our grammar onto the world.

The adjective *abstract* is notoriously used differently among mathematicians and among philosophers, but when speaking to a philosophical audience, I will say that numbers, sets, functions, and other mathematical objects are abstract, expecting to be understood by so saying to be merely repeating the negative assertions —no coordinates, no dates, no mass, no charge— I have just made in other terminology. Now if Xs depend on Ys in any ordinary sense of *dependence*, then Xs cannot have existed at times before Ys existed. But to say that numbers —numbers themselves, as contrasted with various human ideas and concepts of numbers— did not exist at times before people existed is to assign dates to numbers, which I have already ruled out. Hence I will take the characterization of mathematical objects as abstract to include the characterization that they are

not dependent on human beings, in any ordinary sense of *dependence*.

If it becomes clear to me that some philosophers are using ordinary words like *exist* and *depend* in extraordinary ways, I will not affirm that mathematical objects exist and are independent of human beings in *their* senses of those words. Nor will I deny it, since I don't understand those extraordinary senses, and really I am of the opinion that those who are using *existence* in a way having nothing to do with ordinary mathematical criteria for evaluating existence theorems, for instance, ought to give the word back to those of us who want to use it in its ordinary sense. They should introduce some new word for their new sense of the word *exists*, and likewise with the word *dependence*.

One encouraging development since the 1990s is precisely that there seems to be nowadays less use of ordinary words like *existence* and *dependence* in extraordinary senses, and more of a tendency to formulate contentious metaphysical claims in an extraordinary so–called *hyperintensional* vocabulary of *fundamentality* or *grounding* or the like. Such claims are clearly anti–platonist, but they can also be called anti–nominalist, or at least non–nominalist, in the following sense. Since mathematicians do prove results they call *existence theorems*, but none they call *fundamentality theorems* or *grounding theorems*, by switching to the new vocabulary, metaphysicians who want to deny that mathematical objects are fundamental, say, or who want to insist that they are grounded in something else, distinguish themselves from nominalists who wish to deny that mathematical objects exist, since what these metaphysicians are saying does not even give an appearance of contradicting established mathematics or science, and they by saying it do not even give an appearance of engaging in philosophical *Besserwissenschaft*, as contrast with a merely extra–scientific metaphysics. And that is all to the good. I will myself, however, neither endorse nor deny claims about fundamentality or grounding, since the new jargon means nothing to me.

In short, my anti–nominalism consists in affirming that our current mathematics and science is, subject to the usual fallibilist qualifications that are part of science itself, *all right*, and indeed all right when taken more or less at face value. My anti–platonism consists in denying that our current mathematics and science is *uniquely right*, in the sense that extraterrestrials who had a different kind of mathematics and science would be doing something wrong that we are doing right. Our world majority custom of driving on the righthand side of the road is *all right*, though it cannot be said to be *uniquely right* in a sense implying that the British custom of driving on the lefthand side of the road is not all right. Each custom is all right in its place. Likewise with mathematical and scientific theories, I say.

The double–barreled anti–nominalist but anti–platonist attitude seems to me the only scientific attitude to take towards science. A scientific attitude towards science must begin by accepting science, including its mathematical apparatus, which means rejecting nominalistic denials of standard mathematical existence theorems, and rejecting grammatical solecisms that would try to put dates to mathematical objects, and so on. A scientific attitude towards science must also reject platonism, and recognize that our science is not the product of a god or demiurge, but rather a product of certain organisms, ourselves, in a certain environment, the world, and that different organisms in the same environment might produce a science of a different shape, and be none the worse for it. Here anti–platonism and anti–nominalism do not conflict, because they are in the last analysis theses enunciated at different levels of language. Anti–platonism is a meta–theoretic statement about the global or overall status of our science and mathematics as a whole. Anti–nominalism is a reiteration of particular local object–language mathematical existence statements, along with what are obvious consequences thereof if the statements are taken at face value. The reconciliation perhaps in the end turns on little more than a firm insistence on distinguishing mention from use.

§ 3. Contrasting and Opposing Views: Miscellaneous Varieties

So much by way of summary of the view to which I subscribe —and by contrast brief, preliminary notice of another form of anti–platonist non–nominalism, that which is focused on fundamentality and grounding, that has become ever more prevalent of late. I turn next to what has been for me perhaps the most important contrasting view, the sort of anti–nominalism that bases itself on the far–famed *indispensability argument*.

This I doubly reject, or at least, doubly question. On the one hand, I doubt the indispensability claim itself. I doubt the impossibility in principle of dispensing with talk of mathematical objects. As I suggested earlier in discussing exobiology, it probably is possible to dispense with the ordinary kind of talk mathematical objects if one doesn't care too much about the elegance or illuminatingness of whatever kind of talk would replace it. To be sure, I don't doubt the inconvenience in practice of trying to do science without numbers, and I don't doubt that only extraterrestrials with a mental organization quite different from ours could be truly comfortable working with theories that dispense with any mathematical objects.

On the other hand, I reject what the kind of anti–nominalism that rests on indispensability considerations seems to concede to the nominalist side. I reject

the concession that *only* indispensable necessity could justify retention of the kind of mathematical apparatus found in current scientific theories. For me it is sufficient that the employment of such an apparatus has long been customary, and is very convenient. I do not find anything more than that needful to justify the retention of such apparatus. I reject the line of thought that takes it to be obvious that if only we could somehow do without numbers, we should.

There are many other objections or forms of resistance to the line of thought I have long pursued and here sketched. Since, as I have already indicated, few philosophers today are platonists in any historically serious sense, objections to the line I favor, when any notice of that line has been taken at all, have come exclusively from the anti–anti–nominalist side. I won't consider all the objections that are out there. There wouldn't be time to deal with all of them, and some of them seem to me merely to reveal confusions on the part of the objectors. For instance, some professional mathematicians who are also amateur philosophers seem to think that since mathematical objects are surely not physical, they must be mental, shared human ideas and concepts, and so dependent on us. I take it that my audience will have read their Frege know what is wrong with this kind of thinking.

Another popular objection, this one often met with from professional philosophers, strikes me as equally confused. I mean here the line of thought that alleges that there is some major mystery about how we could ever acquire *knowledge* of abstract objects, independent of us, lacking spatiotemporal location, and mass and charge and the rest. If we believe our current mathematically formulated science to be even probably, largely, approximately correct, taken at anything like face value, then we have got a *belief* in theories implying or presupposing the existence of abstract mathematical objects. If one wonders *how* one possibly could acquire such belief, the standard histories of mathematics and science display not only how one possibly could do it, but how we actually have done it. If there is any problem, then, it must concern how such belief could possibly constitute knowledge. What isn't true isn't knowledge, but I take it this is not the issue: I take it that the whole point of raising the issue of knowledge is to avoid having to address directly the issue of truth. I also take it that gettierology is not the issue, either. So the alleged mystery must be about how the actual historical route by which we arrived at our current scientific beliefs could be a route to beliefs that are *justified*. And here I find a confusion, or at least, a need for a distinction.

For it is virtually a tautology that our accepting the theories we have accepted is justified by our scientific standards of justification. We have no other grounds for identifying a standard as one of our scientific standards for theory acceptance

except the historical record of what theories we have actually accepted. Since theories with a substantial mathematical apparatus have been accepted early and late throughout the history of science, there really is no room for doubting that our accepting the mathematicized theories we actually have accepted is justified by scientific standards. Thus there can only be a problem of justification for those who are not viewing the question scientifically but rather are, whether they are aware of so doing or not, imposing some non–, un–, or anti–scientific standards.

It is easy to see how such standards could create problems. If one wanted to insist, for instance, that mathematical objects can only be legitimately admitted if we are aware of them through a faculty analogous to those through which we are aware of physical objects, then indeed there would be trouble. Likewise if one wanted to insist that when two sentences both get into our theory that are parallel in grammar, as with

> The planet Venus has no natural satellites.
> The number seven has no proper divisors.

they must get into our theory in parallel ways. Such an insistence would surely get one into trouble. But what could justify imposing such standards of justification? Certainly not a scientific attitude. From a scientific point of view there can be no room for an epistemological argument for nominalism.

§ 4. Contrasting and Opposing Views: Speculative Ontology

Two other objections seem to me by contrast to require more serious consideration, and less peremptory dismissal, as showing real deficiencies, if not in anti–platonist anti–nominalism as such, at any rate in my brief exposition and defense of it so far. In describing the first of these, let me revert temporarily to theological language, which you are free to take metaphorically. Our theoretical view of the world includes both mathematical and physical objects. The objector holds that I am right that the mathematical objects are not there in a God's–eye view of the world, but insists that physical objects by contrast *are* there in such a view. If the inclusion of mathematical objects in our picture of the world is something that it would probably be practically infeasible for us to do without, that is just one aspect of where our picture of the world is distorted by human weakness; by contrast, the inclusion of physical objects in our picture of the world is a respect in which our picture of the world agrees with God's, or with ultimate metaphysical reality as it is in itself, behind all merely human representations.

Now in my experience quite a few who have tried to express in more literal terms what has just been said in metaphorical theological terms have been led to say that the existence of mathematical objects, unlike that of most physical objects, is dependent on us, a formulation that I have claimed is ultimately a grammatical solecism if *dependence* is taken in any ordinary sense, and of questionable intelligibility if not. The objector may grudgingly grant that I am right to say that something grammatically dubious or problematic is going on, but will insist that I am putting too much weight on grammar. Grammatical pedantry, and hair–splitting and nit–picking about object–language versus metalanguage, should not be allowed to distract us from the important insight that there is a fundamental difference in status between the physical objects we encounter and the mathematical objects we posit.

The objector may not go so far as to say that it is "crime against the intellect" to ignore this alleged difference, as Hart 1977 says in reviewing Steiner 1975, but some may wish to borrow from participants in another controversy (Anderson and Belnap, 1975) the strikingly–formulated objection against those who harp on the use–mention distinction that "such harpists are plucking a metaphysical tune on merely grammatical strings." I myself may be especially reproached for neglecting philosophy seminars on hyperintentionality, the suggestion being that it will most likely be from discussion of just such topics as fundamentality and grounding that a more adequate literal statement of what the difference in status between mathematical and physical amounts to can be expected to emerge. Such is the next objection I wish to consider. I have been straining to express the other side's claim as forcefully and plausibly as I can.

I have found it quite a strain indeed, and need to catch my breath a bit before undertaking to respond. So I will approach the matter at a leisurely pace from an oblique direction. The first thing I might say is that, for me, the objection perfectly illustrates the old aphorism attributed to Lichtenberg, to the effect that *with most people disbelief in one thing is based on blind faith in another.* (*Bei den meisten Menschen gründet sich der Unglaube in einer Sache auf blinden Glauben in einer anderen*, said to be from Notebook L, Aphorism 81, though I have been unable to verify the reference.) In the objection the disbelief is in mathematical objects, and the blind faith in the ultimate metaphysical reality of physical objects, their appearance in a God's–eye view of the world. I will call conjectures and hypotheses about God's–eye views of the world *speculative* metaphysics, in contrast to what Strawson (1959) called *descriptive* metaphysics, concerned with the structure and categories of *our* view of the world, not God's.

The speculative metaphysical objector's profession of faith points to an important omission in my original presentation that I ought to try to remedy. For

while I cited reconstructions that suggest mathematical objects could be dispensed with, I omitted to cite any suggesting that physical objects could be dispensed with equally. To be sure, there already were in the literature, before the nominalist activity of the 1980s to which I did allude, some discussions of alternatives to our usual way of speaking of enduring, extended physical objects. The most notorious was the discussion in Quine (1960a, passim) of how we might avoid talk of *rabbits* in favor of *rabbit–stages* and *undetached rabbit parts*. More recently we have heard talk of avoiding rabbits in favor of elementary particles arranged rabbit–wise, or as I would prefer to say, swarming in rabbit formation. But though I omitted to say anything about the matter in my opening statement, it is possible to go much further than this, and not merely avoid ordinary physical objects in favor of extraordinary ones, but rather avoid all physical objects, ordinary and extraordinary alike, and indeed all objects of any sort, physical and mathematical alike.

Here, continuing the oblique approach, let me begin on this subtopic by going back to an old objection to Descartes (also attributed to Lichtenberg, though this is disputed by Hennig 2018). In English and French and German one says *it is raining* and *il pleut* and *es regnet*, but the pronouns here, though demanded by the grammar of these languages —though not by that of Greek or Latin or Spanish, among others— do not denote any real subject: there is not, besides the raining that is going on, some real subject that is *doing* the raining. Nietzsche (1887/1967, section 13) makes a similar point about lightning flashing. Quine somewhere suggests we might better say, not *it is raining*, but simply *raineth*. In these term, the complaint is that the argument Descartes appeals to in order to establish the *I think* really only establishes *thinketh*. Descartes tries to get two for the price of one, not only the thinking that is going on, but the self that is doing this thinking —assuming a quasi–scholastic metaphysics of *substance* and *mode,* or in more modern diction, *object* and *property*. The suggestion, in any case, is that such a metaphysics is simply a projection onto the world of the *argument–predicate* structure of sentences of our language, the same grammar that demands the *it* or *il* or *es* when we speak of raining.

Now the argument–predicate structure is very widespread, and for all I know universal, among human languages, but even if it is part of universal human grammar, which is as may be, universal human grammar itself is species–specific, and there is no reason to suppose that other intelligent creatures would have a language with all the features universal among human languages. To be sure, it is not easy to imagine what an alien language that did not have an argument–predicate structure would be like, or what an extraterrestrial theory of the world formulated in such a language, and dispensing with any objects whatsoever,

mathematical or physical, ordinary or extraordinary, would be like. And so it cannot be as easy to exhibit an alternative theory without any objects at all as it was to exhibit an alternative theory without any specifically mathematical objects, as the nominalists of the 1980s were attempting to do, with partial success. Nonetheless, there are some hints in the literature, which I have mentioned in passing in the past, and I will mention three very briefly here again, with apologies for the repetition, but with the excuse that the matter really is central to evaluating the merits of my preferred way of reconciling anti–platonism with anti–nominalism.

First, Borges (1962, p. 23) in his well–known tale "Tlön, Uqbar, Orbis Tertius" hints at a noun–free language in which one would say, instead of *the moon rose above the river* something that might be approximated by *upward behind the on–flowing it mooned*. Second, Whorf (1956, p. 215) outright asserts that the Nootka speak just such a language, that the Nootka tongue has only verbs and not nouns, so that one does not say *there is a house* but rather *it houses*, by which I take it he means *houseth*. Whether Whorf is correct about the actual Nootka need not concern us, since we are dealing with questions of possibility, not actuality. And third, the apparatus of *predicate–functor logic*, as in Quine 1960b seems to offer a general means of replacing an argument–predicate or noun–verb with a verb–adverb structure. Sketchy as these suggestions are, I think they give us reason to suspect that if *ontology* is understood to be the study of what sorts of objects have to recognized in in a God's–eye view of the world, then the subject is empty. Such anti–ontologism underlies my preferred way of reconciling anti–platonism with anti–nominalism. The persistence of the speculative metaphysical objection I have just been considering suggests that the case for anti–nominalism can never be made wholly convincing until the case for a more general anti–ontologism has been made.

§ 5. Contrasting and Opposing Views: Structuralist Interpretations

The last objection I wish to consider is just that my assessment that our existing scientific theory of the world involves objects such as the number two, for instance, and that this number two is, as we usually implicitly conceive it, something abstract and independent of human beings, has been made too quickly, and that the situation really is much more complicated. And indeed, though I think my assessment is correct as regards a long enough stretch of the history of a large enough portion of science, the objection is quite right in suggesting that the larger situation is more complex, and deserving of closer attention and more analysis. In personal terms, I was long conscious, for instance,

that my writings on anti–nominalism were postponing a confrontation with so–called structuralism that really would have to be engaged in eventually, but that I have only taken up in a serious way with my latest book (Burgess 2015).

Philosophers in the analytic tradition ought to recognize that it is getting off on the wrong foot to begin an inquiry into the nature of mathematics and its objects by asking *what is the number two?* It is better to go meta–linguistic and ask *what does the numeral* two *denote?* But even that formulation is suboptimal, since it is not self–evident that the use of the numeral is to denote something. It is better, therefore, to ask *how is the numeral* two *used?* But even that formulation is still suboptimal, since it is not self–evident that the numeral has had only one kind of use over the course of history or has only one kind of use today. It is better, therefore, to ask *what are the various ways the numeral* two *has been and is used?*

Here the work of Friederike Moltmann (beginning with Moltmann 2013), on what she calls *natural language ontology* —it might also be termed, recalling Strawson's terminology, *descriptive* as opposed to speculative ontology— suggests that we should not only ask, as I have just done, about the numeral *two*, but also, as a separate question, about the phrase *the number two*, as well as about phrases formed through the *number–of* construction, like *the number of my hands*, even though it is correct to say that the number of my hands is two. For Moltmann demonstrates, with examples from several languages, that these three types of expressions are fairly consistently used in ordinary language today in distinct ways. She has even in private correspondence reminded and informed me —that is, reminded me as regards French and informed me as regards German — that in some languages when one speaks of the number of, say, a taxi–cab, the word used is a special one and not the ordinary word for number: *número* rather than *nombre, Nummer* rather than *Zahl*. In the end a very complicated story, extended over a long period, gradually emerges. I have time only to sketch a few high–lights, mentioning four stages in the developing usage of numerals, with occasional glances at related expressions.

At the first stage, there is no sign of mathematical objects, not even numbers. Numerals appear not as nouns but only as what in traditional grammar would be considered adjectives, as in *two sheep and two sheep make four sheep*. Well, this doesn't just work for sheep, so one may even at this sort of stage say *two and two make four*, while meaning no more than *two things and two things make four things*, whatever sort of things. Here the numerals are still, despite ellipsis, being implicitly used adjectivally.

As to the antiquity of this usage compared with others, and how long it

persisted, I thought I would try checking the Hebrew Bible (though I am obliged to rely on English translations that claim to be literal, plus a peek at the Septuagint). If we look at one of the earlier parts, the Book of Numbers, whose overarching subject is a review, including a counting, of men of military age, we find that the results are not expressed as we would express them, using the *number–of* construction. It is not written that *the number of men reviewed was six hundred thousand*, but rather, *all they who were reviewed were six hundred thousand*, meaning of course *six hundred thousand men*, with the numeral implicitly adjectival. (I owe to my colleague Benjamin Morison the confirmation of my impression that the KJV of Numbers 1:46, "all they that were *numbered* were six hundred thousand…" is an over–translation compared with the LXX, as well as the suggestion that something like "reviewed" or "inspected" would be more accurate than "numbered".)

By the time we get to the late parts, we do find a *number–of* construction. The Book of Ezra begins with Cyrus returning some loot that was taken by Nebuchadnezzar, gold and silver vessels. He has his treasurer *number them unto*, which is to say, *count them out to*, a certain prince, and then we get the results, *this was the number of them, big gold vessels, so many, smaller gold vessels, so many, and so on.* Here *the number of* simply means *the count of*, what you get up to when you count them, a rather different usage from that traced in Moltmann's work on present–day language, it seems to me. But I would hesitate to claim that numbers have been reified, or begun to be treated as objects, solely on the basis of this usage.

At a second stage, numbers are definitely being treated as objects. Sometime between the legendary Pythagoras and the historical Plato we begin to get talk of the number 6 being perfect, the number 7 being prime, the number 8 being a cube, the number 9 being a square, the number 10 being triangular, and so forth. Here we definitely have a nominal, not an adjectival use.

Kneale and Kneale (1963, chapter VI, section 2) discuss a text of Speusippus to show that the new way of thinking and speaking led to some grammatical disruption in a highly–inflected language like Greek (with four different expressions, for instance, ranging from masculine singular to neuter plural, being used in a single text for what for us would be the number ten). And we know that some metaphysical fireworks went off, with speculations about *the one* and *the two*, or the *monad* and *the dyad*. And then there are those hints that the Pythagoreans held some numbers to be male and some to be female, and some pairs to be friendly, for that matter, though we need not suppose all this was meant completely literally.

It is clear, despite the scarcity of documents from the relevant period, that some major intellectual development has taken place, which I would describe as the definitive reification of numbers as objects. What would not be clear without looking beyond classical antiquity is whether the new development was pointing in a scientific or a superstitious direction: Number theory and numerology seem to be intertwined, like astronomy and astrology at the same period. And the mathematics in Greek science, as with Euclid's *Optics* or Archimedes' *Floating Bodies* eventually took a geometric rather than an arithmetic route. *God always geometrizes* and *Let none ignorant of geometry enter here* were the slogans.

A third stage is well–established by the beginning of the modern period and lasts down to the middle nineteenth century, at least, and later in many quarters. Here we find, I would say, that numbers —and indeed numbers of different kinds— are being treated as objects, and as such are playing a role in science. The situation at this period answers best to what I was assuming in my sketch of anti–nominalism.

I note also that the usage of bare numerals like *two* and the fuller expression *the number two* have become in many mathematical contexts more or less interchangeable. Thus one can equally say *two is a prime number* and *the number two is prime*. More than this, even the objects introduced by the *number–of* construction seem to be identified by mathematicians with the number zero, the number one, the number two, and so on. We see this with Euler's totient function ϕ, which plays such a role in elementary number theory. It is a function from the positive integers 1, 2, 3, … to the positive integers, defined by letting $\phi(n)$ be *the number of* numbers $< n$ that are relatively prime to n, meaning, that have no common factors > 1 with n. Indeed, mathematicians pretty clearly take a *numéro* or a *Nummer* to be the same thing as a *nombre* or a *Zahl*. (The reader can check this by Googling to find French and German translations and paraphrases and summaries on–line of the well–known anecdote about Ramanujan, Hardy, and taxi–cab number 1729.) The delicate distinctions traced by Moltmann, though perfectly real in everyday usage, seem largely to get lost in the usage of mathematicians.

A fourth stage is incipient among professional mathematicians by the late nineteenth century, when we get the beginnings of the developments brought to the attention of philosophers by the classic Benacerraf 1965 —developments which have led to contemporary structuralism with its sometimes extravagant ontological claims about numbers. The story is a complicated one, and my analysis of it takes up quite a few pages (pp. 123 ff) in my recent book. I have not the space to recapitulate here, so let me just say that I see the puzzling aspects of

the situation as due to two historical developments, Dedekind's rigorous axiomatization of arithmetic, and the later project, most visible in the Bourbaki encyclopedia, of incorporating the whole of mathematics in a single axiomatic framework. At work are two principles. One I call the *paradox of rigor*. This is the observation that *a completely rigorous treatment of a given subject matter ceases to be a treatment of that subject matter*. The other I call the *paradox of foundations*. This is the observation that *a foundation for the whole of mathematics must fail to be a foundation for some of its parts*.

Both the history and the principles to which I have given a paradoxical formulation would require considerable elaboration, taking into account the real as well as the natural numbers. This I have tried to supply in my book, but there can be no question here of my even summarizing the complicated story. All that matters for my reply to the last of the objections I have been considering is that though present–day mathematics and therewith present–day physics cannot quite be taken at face value, as I assumed could be done throughout the bulk of my discussion, correcting to take into account historically late, highly sophisticated developments among professional pure mathematicians still leaves us with a science full of abstract objects, and the basic picture is unchanged. Thus in the end I see in the objections canvassed no reason to abandon or compromise anti–platonist anti–nominalism, which I continue to commend to your consideration.

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Reconciling Anti–Nominalism and Anti–Platonism in Philosophy of Mathematics

The author reviews and summarizes, in as jargon–free way as he is capable of, the form of anti–platonist anti–nominalism he has previously developed in works since the 1980s, and considers what additions and amendments are called for in the light of such recently much–discussed views on the existence and nature of mathematical objects as those known as hyperintensional metaphysics, natural language ontology, and mathematical structuralism.

Keywords: Nominalism · Platonism · Ontology · Structuralism

La reconciliación del antinominalismo y antiplatonismo en la filosofía de la matemática

El autor repasa y resume, usando un lenguaje tan libre de tecnicismos como sea posible, la forma de antiplatonismo antinominalismo que él ha desarrollado previamente en su obra desde los años de 1980, y reflexiona acerca de cuáles adiciones y enmiendas se requieren a la luz de puntos de vista muy discutidos recientemente sobre la existencia y la naturaleza de los objetos matemáticos, tales como los que se conocen como metafísica hiperintensional, ontología del lenguaje natural y estructuralismo matemático.

Palabras Clave: Nominalismo · Platonismo · Ontology · Estructuraismo.

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