Some notes on the Aristotelian doctrine of opposition and the propositional calculus

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§1. Introduction

GIVEN ANY TWO FORMULAS A AND C IN STANDARD PROPOSITIONAL LOGIC, we can condition A to depend on the truth values of C. For this porpoise, we examine the rows of the truth table of C that are true (if any) separately from the rows that are false (if any). If A is independent of C wherever C is true (there are two different truth distributions), we replace the corresponding values of A with those of C. We do the same in the rows where C is false. We leave the rows where A depends on C unchanged, if they are not all. On the other hand, if A depends on C in all rows, both those in which C is true and those in which it is false, we simply replace A with C. (This prevents a formula from being a logical consequence of its own negation: McCall, 2012; Wansing, Summer 2022.) By the previous method, we condition the truth values of C on the truth values of A. The resulting formula is logically equivalent to C, a tautology, or a contradiction. We write A/C to consider A under the condition C as described. It is easy to see that the relation "/" is reflexive, symmetric, and transitive.

An example of the application of the above informal idea is as follows. Given the propositional variables *s* (0 1 1 0) and *u* (0 0 1 1), the formula (¬s ↛ u) (1 0 0 0) under the condition *s* would be

$$((\neg s \nrightarrow u) \,/\, s)\ (0\ 0\ 0\ 0).$$

The formulas include the truth values in parentheses, where 1 means true while 0 means false. As a result of the conditioning, the initial sentence loses its independence of the condition where it was. In the proposed example, the affected rows (first and

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Disputatio. Philosophical Research Bulletin
Vol. 12, No. 26, Dec. 2023, pp. 53–70
ISSN: 2254–0601 | [EN] | **PAPER**

fourth) all became false. From now on, we will underline the corresponding truth values to make them more visible, preserving their conventional meaning. We will generally speak of "the conditioned truth table of A given C" or "the truth table of A under the condition C".

The conditioning of propositions applies to the "paradoxes of material implication." E.g., the sentence "If 2 + 2 = 5, then 2 + 2 = 4" is a logical truth—contrary to what intuition suggests—because the antecedent is false. However, when we represent "2 + 2 = 5" by *s* and "2 + 2 = 4" by *u*, we know that *s* is false while *u* is true, so we should rewrite the formula (s → u) (1 0 1 1) under the condition (¬s ^ u) (0 0 0 1) to get

$$((s \to u) / (\neg s \wedge u))\ (\underline{0}\ \underline{0}\ \underline{0}\ 1).$$

It is no longer true as long as the antecedent is false; in fact, it is true only when ¬*s* and *u* are true simultaneously. The latter expression is logically equivalent to (u ↛ s) (0 0 0 1), which cannot be misinterpreted. The sentence is now plain in natural language: "From 2 + 2 = 4, it does not follow that 2 + 2 = 5".

In this paper, we develop some of Williamson's ideas on the contribution of propositional calculus to a better understanding of Aristotelian logic (Williamson, 1971; 1972; 1988). Specifically, the use he makes of truth tables in analyzing the structure of the traditional square of opposition is enhanced by the conditioning technique just shown. The interaction between different propositions allows us to establish conditioned truth tables. Taking advantage of this possibility, we propose a new reading of various passages of the *Organon* related to opposition.

The formalization that we adopted will compromise the interpretation of Aristotelian logic. The quantified expressions and the relationships between terms, which characterize syllogistic reasoning, explain that predicate calculus and class logic have dominated approaches to matter in symbolic logic. The resources of modal logic get weighted when necessary. However, there are several Aristotelian texts open to alternative formalizations. A recurring theme in the *Organon* is the study of the forms of logical opposition that exist between all kinds of propositions. Analyses are not limited to sentences with quantified expressions. Although the list of academic papers is extensive (as far as the square of opposition is concerned, there is a series of monographic publications of almost annual periodicity: Béziau & Payette (eds.), March 2008; Béziau & Payette (eds.), 2011; Béziau & Jacquette (eds.), 2012; Beziau & Read (eds.), 2014; Béziau & Giovagnoli (eds.), June 2016; Béziau & Basti (eds.), 2017; Béziau

& Stamatios (eds.), 2017; Béziau & Lemanski (eds.), March 2020; Béziau & Vandoulakis (eds.), 2022, among others), there is still a gap. It is unusual to approach the subject using propositional calculus.

§ 2. Williamson's squares of opposition and the conditioning of propositions

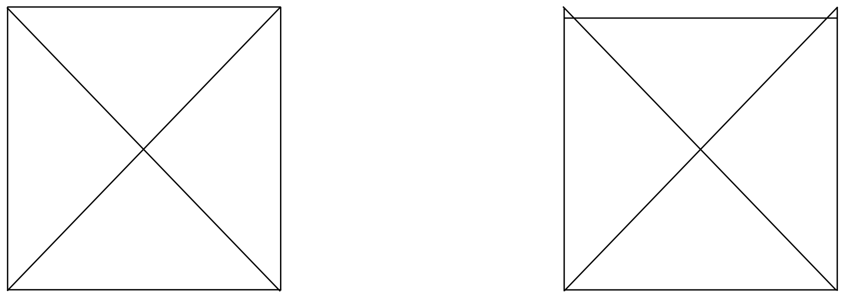
Williamson convincingly defended the coincidences between syllogistic and propositional logic. One of his most important contributions was demonstrating that there were two plausible variants of the traditional square of opposition in propositional calculus and that there should also be two in the syllogistic. The truth tables reveal that the number of opposable formulas is higher than had been assumed (Williamson, 1972: 498–499). The conditioning of the propositions adds new possibilities to the study of opposition. Adopting a hypothesis affects the truth tables of the dependent formulas, multiplying the identifiable similarity and dissimilarity relationships. In this way, we learn more about the internal structure of the diagrams of opposition.

We use two propositional variables (*s* and *u*) in the formalization of some passages of the Aristotelian *Organon*, as well as the connectives negation (¬), material implication (→), material nonimplication or abjunction (↛), biconditional (↔) and exclusive disjunction (↮). Technically, the novelty lies in the idea of conditioning. The truth tables of the formulas are altered in various ways by the assumption introduced. The sixteen logical functions for two propositional variables are present through the conditioning symbol (/). Different connectives can provide the same conditioned truth table in each case. Underlining serves as a visual marker of the conditioning of propositions, but the truth values retain their usual meaning. Therefore, conditioned formulas can be combined using logical connectives with absolute normality. With the noted exception, propositional calculus and truth tables fall into the familiar traditional way. A more formal definition is unnecessary.

Williamson (1972: 497–499) considers four pairs of opposite formulas in the corners of his squares of opposition. They are as follows, based on the variables *s* (0 1 1 0) and *u* (0 0 1 1), and in the connectives already mentioned: (s → u) (1 0 1 1), (s ↛ u) (0 1 0 0), (s → ¬u) (1 1 0 1), (s ↛ ¬u) (0 0 1 0), (¬s → ¬u) (1 1 1 0), (¬s ↛ ¬u) (0 0 0 1), (¬s → u) (0 1 1 1), and (¬s ↛ u) (1 0 0 0).

The two squares are shown in Figure 1:

(s ↛ u) (0 1 0 0)   (s ↛ ¬u) (0 0 1 0)   (¬s ↛ ¬u) (0 0 0 1)   (¬s ↛ u) (1 0 0 0)



(s → ¬u) (1 1 0 1)   (s → u) (1 0 1 1)   (¬s → u) (0 1 1 1)   (¬s → ¬u) (1 1 1 0)

Figure 1: Williamson's squares of opposition adapted to the propositional variables s (0 1 1 0) and u (0 0 1 1), as well as to the connectives →, ↛, and ¬

The introduction of conditions in the above formulas will lead to several variants of the diagrams of opposition. We will limit ourselves to presenting those necessary for commenting on some selected texts of Aristotle. We want to make visible an effect of the choice of conditions. Perhaps it is possible to interpret the conditioning of a complex formula as its decomposition into simpler components.

In reviewing the traditional concept of opposition, we will employ six conditions: s (0 1 1 0), ¬s (1 0 0 1), u (0 0 1 1), ¬u (1 1 0 0), (s ↔ u) (1 0 1 0), and (s ↮ u) (0 1 0 1). These are all possibilities with two true rows in the truth tables. Under the right conditions, the relationships of similarity and dissimilarity between the squares' formulas emerge clearly. E.g., using the conditions *u* and ¬*u*, and the biconditional with a sense of identity, we can construct tautologies such as the following:

(((s ↛ ¬u) / u) ↔ ((s → u) / u)) ≡ (((s → ¬u) / ¬u) ↔ ((s ↛ u) / ¬u)) ≡ (((s ↛ u) / u) ↔ ((s ↛ ¬u) / ¬u)) ≡ (((s → ¬u) / u) ↔ ((s → u) / ¬u)).

With the same conditions, we can form a new group of tautologies in which the exclusive disjunction is an indicator of contradiction:

$$(((s \nrightarrow u) / u) \nleftrightarrow ((s \rightarrow \neg u) / u)) \equiv (((s \nrightarrow \neg u) / \neg u) \nleftrightarrow ((s \rightarrow u) / \neg u)) \equiv (((s \rightarrow \neg u) / \neg u) \nleftrightarrow ((s \rightarrow u) / u)) \equiv (((s \nrightarrow u) / \neg u) \nleftrightarrow ((s \nrightarrow \neg u) / u)) \equiv (((\neg s \rightarrow \neg u) / \neg u) \nleftrightarrow ((\neg s \rightarrow u) / u)).$$

We can do the same with the other two pairs of conditions (s and ¬s, and $s \leftrightarrow u$ and $s \nleftrightarrow u$).

§ 3. The presence of underlying conditions in Prior Analytics Chapter 46

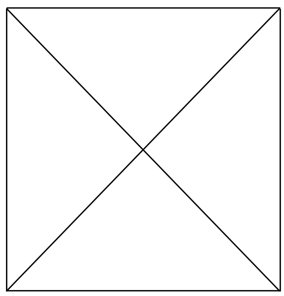
As we have seen, when a certain condition is assumed, it is possible to examine what effect it has on the truth table of the formulas that depend on it. One good place to test this idea is Chapter 46 of *Prior Analytics*. When analysing the opposition between affirmations and denials, Aristotle employs a diagram that served as the basis for developing the traditional square of opposition:

> Let A stand for ‘to be good,’ B stand for ‘not to be good,’ C stand for ‘to be not good’ (which is below B), and D stand for ‘not to be not good’ (which is below A). Now, either A or B will belong to everything but not <both> to any same thing; and also, either C or D will belong to everything, but not < both> to any same thing. (Smith translation, 1989).

The letters A, B, C, and D serve as declarative propositions. Smith (1989: 178) has proposed treating affirmations and denials as “pairs of *open sentences*: sentences in which the subjects are only variables.” But because of the way truth and falsehood articulate oppositions in the text (sometimes explicitly), formalization seems to require compound propositions. Different conditions provide alternative readings of them, giving more accurate information about the truth values. Aristotle illustrates his arguments with the examples “to be good,” “a thing is white,” and “a log is white.” All logical formalization inevitably entails assumptions. Since the priority is to identify the truth tables underlying the oppositions, we must take the license to add “is” or “is not” to all meaningful expressions, giving them a propositional status (*On Interpretation* 16a17-17a17 and 20b31-21a4; *Categories* 1b25-2a12). We will represent by *s* the simple proposition “it is a thing” (or, in its case, “it is a log”), and by *u* “it is good” or “it is white.” (If the formalization of the examples is doubtful, they are dispensable: the central theses are reworked at the end of the chapter, acquiring a general tone independent of any illustration.) Connectives can, in principle, reflect affirmations and denials. The letters A, B, C, and D correspond to the propositions (s → u), (s ↛ u), (s → ¬u), and (s ↛ ¬u), respectively. Before the changes that may be implied by the condition we assume in each case, the material implication serves as a connective in affirmations, and the material nonimplication or abjunction in denials. B is the negation of A [B ↮ A], and D is the negation of C [D ↮ C]. This gives us one of the Williamson squares:

A B

(s → u) (1 0 1 1) (s ↛ u) (0 1 0 0)



(s ↛ ¬u) (0 0 1 0) (s → ¬u) (1 1 0 1) D

C

Figure 2: Diagram of Chapter 46 of the Prior Analytics

It is convenient to have Figure 2 in view to interpret the indications that Aristotle gives about the structure of his diagram. Generally, it is sufficient to recognize the logical conditions implicit in the relationships between formulas A, B, C, and D. These conditions are, as the case may be, *u* or ¬*u*. We insert in square brackets in Aristotle's text the formalizations and appropriate explanations:

> And it is necessary for B to belong to everything to which C belongs: for if it is true to say that something is not-white [((s → ¬u) / ¬u) (1 1 $\underline{0\ 0}$)], then it is true to say that it is not white [((s ↛ u) / ¬u) ($\underline{1\ 1}$ 0 0), this formula is logically equivalent to the previous one] (for it is impossible to be white and to be not-white at the same time [(((s → u) / u) ↮ ((s → ¬u) / ¬u)) (1 1 1 1), exclusive disjunction points to a contradiction], or to be a not- white log and to be a white log), so that if the affirmation does not belong then the denial will [if it is contradictory that ((s → ¬u) / ¬u) (1 1 $\underline{0\ 0}$) is fulfilled at the same time as the affirmation ((s → u) / u) ($\underline{0\ 0}$ 1 1), this latter must be replaced by its denial ((s ↛ u) / ¬u) ($\underline{1\ 1}$ 0 0)]. But C

> does not always belong to B (for what is not a log at all [((s ↛ u) / u) ($\underline{0}$ $\underline{0}$ 0 0)] will not be a not-white log [((s → ¬u) / u) (1 1 $\underline{1}$ $\underline{1}$); the contradiction with the previous formula arises by changing the conditions initially set for C and B]). Therefore, in reverse order, D belongs to everything to which A belongs: for either C or D belongs to everything, and since it is not possible to be at once not-white and white, D will belong to A (for of that which is white [((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1)] it is true to say that it is not not- white [((s ↛ ¬u) / u) (0 0 $\underline{1}$ $\underline{1}$)]). But A will not be true of every D (for it is not true to say A–that it is a white log [((s → u) / ¬u) ($\underline{1}$ $\underline{1}$ 1 1)]– of what is not a log at all [((s ↛ ¬u) /¬u) (0 0 $\underline{0}$ $\underline{0}$)]; consequently, it is true to say D, but it is not true to say A, i.e., that it is a white log). It is also clear that A and C cannot belong to anything the same [((s → u) / u) ↮ ((s → ¬u) / ¬u)) (1 1 1 1)], and that B and D can belong to the same thing [(((s ↛ u) / u) ↔ ((s ↛ ¬u) / ¬u)) (1 1 1 1)].

The conditioned formulas, which are contradictory, are connected by the exclusive disjunction. If they are logically equivalent, the biconditional signals an identity. With these tools, Aristotelian reasoning leads to a formulation of the law of contraposition: If (given the condition ¬*u*) C follows B, then (under the condition *u*) the contrary of B, which is A, will follow the contrary of C, which is D. And conversely, if (by assuming *u*) B does not follow C, then (under the condition ¬*u*) D does not follow A either. The last lines of the passage, about the diagonals of the square, seem to be related to the distinction between affirmations and denials.

Aristotle collects more complex examples of oppositions that adapt to the same diagram's structure. He considers the notions of equality and inequality:

> Privations also have the same relationship to their predications when put in this arrangement. Let A stand for equal, B stand for not equal, C stand for unequal, D stand for not unequal.

Knowing that the *unequal* is an affirmation and what *is not equal* is a negation helps us understand what Aristotle meant at the beginning of the chapter when he specified that in the *unequal* "a subject underlies," but in what *is not equal*, no. ("Nor are 'to be not equal' and 'not to be equal' the same: for there is a certain subject for 'to be not equal,' that is, the thing, which is unequal, whereas there is not any subject for the other. It is for this reason that not everything is equal or unequal [(((s → u) / ¬u) ↔ ((s → ¬u) / u)) (1 1 1 1)], though everything is equal or not equal [(((s → u) / ¬u) ↮ ((s ↛ u) / u)) (1 1 1 1) and (((s → u) / u) ↮ ((s ↛ u) / ¬u)) (1 1 1 1)].") The "underlying subject" corresponds to the formula that has by itself truth values 1 in some rows of its truth table, and not as a consequence of the accepted condition. This occurs in affirmations [((s → u) / ¬u) ($\underline{1}$ $\underline{1}$ 1 1) and ((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1)], but not in denials [((s ↛ u) / ¬u) ($\underline{1}$ $\underline{1}$ 0 0) and ((s ↛ u) / u) ($\underline{0}$ $\underline{0}$ 0 0)].

More complicated is the last example with which Aristotle concludes his examination of propositions to be taken correctly as contradictory. He considers some expressions with quantifiers, but without completing their adaptation to the diagram:

> In the case of a group of things, where the same term belongs to some of them and not to others, the denial might similarly be true either in that they are not all white or in that each of them is no white, though it is false that each is not-white or that all are not-white. In the same way also, the denial of ‘every animal is white’ is not ‘every animal is not- white’ (for both are false), but ‘not every animal is white’.

To formalize quantifier expressions without resorting to predicate logic is a great difficulty. One possibility, to which further work should be devoted, is to examine the rows of the conditioned truth tables corresponding, in the examples, to the items “not- white” (first and second) and “white” (third and fourth). Suppose that “the white is” (*u*). Under this condition, “all are white” can be formalized as $((s \rightarrow u) / u)$ ($\underline{0\ 0}$ 1 1) since the rows corresponding to “white” have truth value 1 and those corresponding to “not white” have truth value 0. Following the thread of the text, the denial of the universal affirmation is not that “all are not-white” which rests on the condition “the not-white is” ($\neg u$). The negative particular we are looking for is “they are not all white”: $((s \nrightarrow u) / u)$ ($\underline{0\ 0}$ 0 0), which at the same time implies that “they are not all not-white.”

We propose to explain conditions as indeterminate propositions (with no universal or particular indication, *Prior Analytics* 24a20) and conditioned formulas as categorical propositions. This idea leads to a curious interpretation of the conditioned formulas whose truth tables are (1 1 1 1) and (0 0 0 0). “Some are not-white and some are white” is opposed to “some are not not-white and some are not white.” A difficulty raised by Aristotle when he states that “‘to some’ and ‘not to some’ are only opposites verbally” (*Prior Analytics* 63b24-30). And certainly, what is white in one formula is not not-white in the other; and what is not white in the latter is not-white in the former. With this formalization, the reader will have a better understanding of the conditioning rules at the beginning of the paper.

The passage continues with a digression on how to work with contradictory categorical propositions in demonstrations. We will not reproduce it. Aristotle then goes on to formulate and universally demonstrate relationships similar to those he had previously shown by examples. He includes in the hypotheses the relationships that occur on one of the diagonals, deducing the consequences for the other diagonal. To avoid contradictions in the text, scholars traditionally swap the position of the letters C and D in the diagram (e.g., Mueller: 2014, 112–114, 132 n. 263; Striker: 2009, 246.) But the apparent inconsistencies are not such when we correctly select and combine the assumptions in the formalization. Conditions are always the touchstone; they generate the conditioned truth tables of the formulas:

> Without qualification, whenever A is so related to B that it is not possible for them to belong to the same thing at the same time but of necessity one or the other of them belongs to everything [that is, they always relate to each other in a contradictory way and always under contrary assumptions: $(((s \rightarrow u) / u) \nleftrightarrow ((s \nrightarrow u) / \neg u))$ (1 1 1 1) and $(((s \rightarrow u) / \neg u) \nleftrightarrow ((s \nrightarrow u) / u))$ (1 1 1 1)], and C and D, in turn, are likewise related [$(((s \rightarrow \neg u) / \neg u) \nleftrightarrow ((s \nrightarrow \neg u) / u)$ (1 1 1 1) and $(((s \rightarrow \neg u) / u) \nleftrightarrow ((s \nrightarrow \neg u) / \neg u)$ (1 1 1 1)], and A follows C [$(((s \rightarrow u) / \neg u) \leftrightarrow ((s \rightarrow \neg u) / u))$ (1 1 1 1)] and does not convert with it [by exchanging the conditions, a contradiction arises: $(((s \rightarrow u) / u) \nleftrightarrow ((s \rightarrow \neg u) / \neg u))$ (1 1 1 1)], then D will follow B [$(((s \nrightarrow \neg u) / \neg u) \leftrightarrow ((s \nrightarrow u) / u))$ (1 1 1 1)] and will not convert with it [$(((s \nrightarrow \neg u) / u) \nleftrightarrow ((s \nrightarrow u) / \neg u))$ (1 1 1 1)]. Also, it is possible for A and D to belong to the same thing [they are logically equivalent under the same condition *u*; $(((s \rightarrow u) / u) \leftrightarrow ((s \nrightarrow \neg u) / u))$ (1 1 1 1)] but not possible for B and C [under the same condition *u*, they are contradictory: $(((s \nrightarrow u) / u) \nleftrightarrow ((s \rightarrow \neg u) / u))$ (1 1 1 1)].

In the general formulation of the "theorem", the hypothesis are satisfied under conditions $\neg u$ or u, as the case may be, and the same goes for the consequences deduced. The above formalization in square brackets facilitates the understanding of the Aristotelian proof:

> First, then, it is evident from the following argument that D follows B. Since one or the other of C and D belongs to everything of necessity [they are related to each other under contrary assumptions in a contradictory manner], but it is not possible for C to belong to what B does (because C brings along with it A, and it is not possible for A and B to belong to the same thing), it is evident that D will follow B [B and C relate to each other under the same hypothesis; when they are contradictory, the contrary of C, which is D, will be equivalent to B].
>
> Next, since C does not convert with A [are contradictory to each other given certain conditions], but either C or D belongs to everything, it is possible for A and D to belong to the same thing. However, this is not possible for B and C, because A follows along with C (for something impossible results [if A and D are logically equivalent under a given condition, B and C will be contradictory under the contrary assumption]). It is evident, then, that B does not convert with D either [they will contradict each other under the respective assumptions that made C and A contradictory], since it is possible for D and A to belong co something at the same time.

If A (conditioned by $\neg u$) follows C (determined by u), then the contrary of C (D under condition $\neg u$) will follow the contrary of A (B under condition u). In reverse order, if C (conditioned by $\neg u$) does not follow A (determined by u), then the contrary of A (B under condition $\neg u$) will not follow the contrary of C (D under condition u). It is a new formulation of the law of contraposition. Identifying the assumptions under which to interpret the formulas is helpful, methodologically, for the correct understanding of

Aristotelian reasonings.

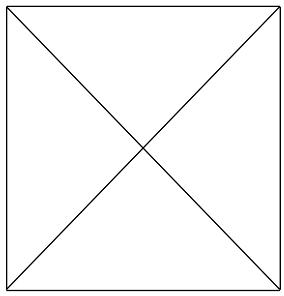
We will not dwell on the final paragraphs of Chapter 46. They show the contradictions to which it leads by erroneously distributing the terms in the diagram. Knowing that A follows C, we err in assigning D the role of affirmation and B that of denial. We will falsely believe that B follows D, i.e., that the contrary of A follows the contrary of C, in breach of the law of contraposition. It is possible that in these lines Aristotle is also suggesting how to include a third sentence in addition to *s* and *u* in the oppositions.

The diagram of Figure 2 appears in *On Interpretation* 19b25–32, as noted in the text itself, which refers the reader to *Analytics*. Aristotle examines the affirmations and negations that the same verb can generate when joined to two different terms. (The example with which he illustrates the diverse possibilities is "a man is just.") A few lines later (19b36–20a2), he completes the study of the opposition in another diagram:

> There are others [opposites] if something is added to 'not-man' as a sort of subject, thus:
>
> 'a not-man is just' 'a not-man is not just'
>
> 'a not-man is not not-just' 'a not-man is not-just'
>
> There will not be any more oppositions that these. These last are a group on their own separate from the others, in that they use 'not-man' as a name (Ackrill translation, 1963).

The formalization of this group of opposites in propositional logic coincides with the formulas of Williamson's square of opposition that were still to be used (Figure 1). Keeping the same propositional variables that we have been using, ¬*s* means "it is something that is not a man" or "it is a not-man" ("not-man" is taken "as a sort of subject" and "as a name"), and *u* means "it is just":

$(\neg s \rightarrow u)$ (0 1 1 1) $(\neg s \nrightarrow u)$ (1 0 0 0)



$(\neg s \nrightarrow \neg u)$ (0 0 0 1) $(\neg s \rightarrow \neg u)$ (1 1 1 0)

Figure 3: Diagram of On Interpretation, 19b25- 32

In the new diagram of opposition, the law of contraposition is also observed in the same terms as before.

§ 4. Oppositions in Chapter 2 of the *Categories*. Nonsubstantial particulars

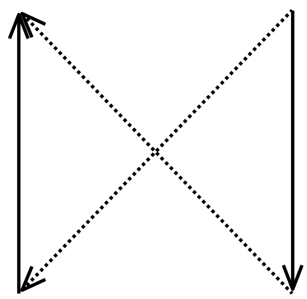
In Chapter 2 of the *Categories*, the logical expressions opposed refer to "kinds of beings." Once again, we give propositional value to Aristotelian terms: *s* represents the "being" of something, *u* is its "individual" character, and ¬*u* its "plural" one. It is probable the influence of the dialogue *Parmenides*, in which Plato analyses the relations between the forms of the One, the Not-One, the Being, and the Non-Being. In the Aristotelian passage, the latter form is missing. Only *u* and ¬*u* serve as conditions. Opposite forms are a central element of the Platonic method: it is necessary "to examine the consequences that follow from the hypothesis, not only if each thing is hypothesized to be, but also if the same thing is hypothesized not to be" (*Parmenides* 135e-136a: Allen translation, 1997). In his analysis of the relationships between forms, Plato successively varies the opposite conditions he assumes and thoroughly investigates their consequences until all logical possibilities have been exhausted.

It is a pity that Aristotle does not expressly indicate which the underlying diagram of opposition is in this chapter of the *Categories*. Those shown in Figure 1 share the same structure and appear to meet the required conditions. We opt for the first of these simply because Aristotle speaks of “things there are” (*s*). In the formalization, the conditions *u* and ¬*u* must be opposed. If the conditioned formulas are logically equivalent to these conditions, “what is present in something” is distinguished from “what is not present in something.” And if the conditioned formulas are tautologies or contradictions, “what is said of something” is distinguished from “what is not said of something.” Again, the use of biconditional and exclusive disjunction is the basis for the identity or contradiction between formulas.

We will reproduce Aristotle's text, minimally separating the possible situations in the order in which he classifies them. Square brackets show the formalization of the examples used as illustrations. The underlying diagram of opposition explains the rearrangement of the four types of entities based on logical criteria (*cf*. Studtmann, Spring 2021):

Primary substance / What is not a primary substance

$(s \rightarrow u)$ (1 0 1 1) / $(s \nrightarrow u)$ (0 1 0 0)



$(s \nrightarrow \neg u)$ (0 0 1 0) / $(s \rightarrow \neg u)$ (1 1 0 1)

What is not a secondary substance / Secondary substance

Figure 4: Diagram of Chapter 2 of Categories

As confirmed in the text, Aristotle establishes two alternative chains of relationships. Both lead from what is not a primary substance to a primary substance, but through a different medium:

(i) A secondary substance (or secondary being) *is said of* a primary substance (or primary being) but *is not present in* it:

> Of things there are: some are *said of* a subject but are not *in any* subject. For example, man $[((s \rightarrow \neg u) / u)\ (1\ 1\ \underline{1\ 1})]$ is said of a subject, the individual man $[((s \rightarrow u) / \neg u)\ (\underline{1\ 1}\ 1\ 1)]$ but is not in any subject $[(((s \rightarrow \neg u) / \neg u) \nleftrightarrow ((s \rightarrow u) / u))\ (1\ 1\ 1\ 1)]$ (Ackrill translation, 1963).

(ii) What is not a secondary substance *is present in* a primary substance but *is not said of* it. In the examples, the "soul" and "body" spoken of are "individual":

> Some are in a subject but are not said of any subject. (By 'in a subject' I mean what is in something, not as a part, and cannot exist separately from what it is in.) For example, the individual knowledge-of-grammar $[((s \nrightarrow \neg u) / u)\ (0\ 0\ \underline{1}\ \underline{1})]$ is in a subject, the soul $[((s \rightarrow u) / u)\ (\underline{0\ 0}\ 1\ 1)$; the subject condition corresponds to the formula that has truth values 1 on its own, without the condition], but is not said of any subject $[((s \nrightarrow \neg u) / \neg u) \nleftrightarrow ((s \rightarrow u) / \neg u))\ (1\ 1\ 1\ 1)]$; and the individual white is in a subject, the body (for all color is in a body), but is not said of any subject.

(iii) What is not a primary substance *is present in* a secondary substance; the text omits that it *is not said about* it. (In the example, the "soul" spoken of is now "universal.") And

(iv) what is not a primary substance *is said of* what is not a secondary substance; no insert that it *is not present in* it:

> Some are both said of a subject and in a subject. For example, knowledge $[((s \nrightarrow u) / \neg u)\ (\underline{1}\ \underline{1}\ 0\ 0)]$ is in a subject, the soul $[((s \rightarrow \neg u) / \neg u)\ (1\ 1\ \underline{0}\ \underline{0})$; it is implied that knowledge *is not said of* the "universal" soul: $(((s \nrightarrow u) / u) \nleftrightarrow ((s \rightarrow \neg u) / u))\ (1\ 1\ 1\ 1)]$, and is also said of a subject, knowledge-of-grammar $[(((s \nrightarrow u) / u) \leftrightarrow ((s \nrightarrow \neg u) / \neg u))\ (1\ 1\ 1\ 1)$; Aristotle does not specify that knowledge *is not present in* knowledge-of-grammar: $(((s \nrightarrow u) / \neg u) \nleftrightarrow ((s \nrightarrow \neg u) / u))\ (1\ 1\ 1\ 1)]$.

Finally, Aristotle observes that in some combinations of the diagram, no subject is possible:

> Some [things] are neither in a subject nor said of a subject, for example, the individual man or individual horse –for nothing of this sort is either in a subject or said of a subject. Things that are individual and numerically one $[((s \rightarrow u) / u)\ (\underline{0}\ \underline{0}\ 1\ 1)$ and, as stated below, $((s \nrightarrow \neg u) / u))\ (0\ 0\ \underline{1\ 1})$; both formulas are logically equivalent to u $(0\ 0\ 1\ 1)]$ are, without exception, not said of any subject, but there is nothing to prevent some of them from being in a subject–the individual knowledge-of-grammar is one of the things in a subject [the individual soul $((s \rightarrow u) / u)\ (\underline{0\ 0}\ 1\ 1)]$.

The text offers some resistance to identifying a logical criterion that explains what operates as a "subject" in each case. That role falls on the affirmative formula, i.e., the primary substance or the secondary substance: The unconditioned formulas have truth values 1 in the rows corresponding to *u* and ¬*u*, respectively. But, given the condition ¬*u*, Aristotle admits as "a merely linguistic subject" what is not a secondary substance. He does not provide any justification from a logical perspective. A reasonable explanation derives from the conclusions of Chapter 46 of *Prior Analytics*. It consists of a reformulation of the law of contraposition: If C (the secondary substance under condition *u*) is said of subject A (the primary substance under condition ¬*u*), then the contrary of A (B, what is not a primary substance under condition *u*) will be said of the contrary of C (D, what is not a secondary substance under condition ¬*u*), which will act as a subject.

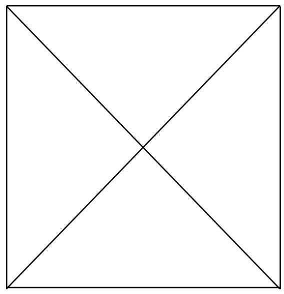
With the described formalization, it is possible to intervene in the controversy about the existence or not of nonsubstantial particulars (Ackrill, 1963; Owen, 1965; Frede, 1987; Matthews, 1989; Devereux, 1992; Wedin, 1993; and Aranyosi, 2004, within a vast list). If the condition ¬*u* (plurality) is assumed, what is not a secondary substance is "a merely linguistic subject." The conditioned truth table contains only the value 0: $((s \nrightarrow \neg u) / \neg u)$ $(0\ 0\ \underline{0}\ \underline{0})$. And if the condition *u* (individual) is assumed, what is not a secondary substance "cannot exist separately from what it is in," i.e., the primary substance. The truth value 1 in the rows corresponding to *u* is not its own but comes from the assumed condition $(((s \nrightarrow \neg u) / u)$ $(0\ 0\ \underline{1}\ \underline{1}))$. Nor does the extralinguistic existence of nonsubstantial particulars appealing to the condition *s* $(((s \nrightarrow \neg u) / s)$ $(0\ \underline{1}\ \underline{1}\ 0))$ seem defensible. Prior to conditioning, what is not a secondary substance was also independent of *s* (the same as was the case with the assumption *u*).

§ 5. The diagram of opposition for *Topics* II, 7

A curious variant of the diagram of opposition appears in *Topics* II, 7. The case at hand, concerning the obligations that friendship entails, Plato treated extensively in the dialog *Lysis* as part of the study of the opposing forms of Likeness and Unlikeness. Aristotle employs only affirmative propositions. However, he had considered denials ("not to harm our friends" and "not to do good to our enemies") in a preliminary discussion of the same case in *Topics* I, 10. The morally desirable actions have to be similar to each other, opposing the rejectable ones that are similar on their side.

Once again, if we reduce the Aristotelian terms to propositions of sentential calculus, truth and falsehood articulate oppositions; “to do good” can be subsumed in *u*, and “to be friends” in *s*. The resulting diagram is as follows:

$(s \rightarrow u)$ (1 0 1 1) $(s \rightarrow \neg u)$ (1 1 0 1)



$(\neg s \rightarrow u)$ (0 1 1 1) $(\neg s \rightarrow \neg u)$ (1 1 1 0)

Figure 5: Diagram of Topics II, 7

It is now easy to obtain a logical formalization faithful to the Aristotelian text: The assumptions under which we interpret the formulas, allow us to identify the logical oppositions that produce contradictions and those that do not. When two opposites of the diagram are equivalent under the same condition and also under its contrary, there is no contradiction. This latter requires the use of contrary assumptions in the two opposite formulas:

> The first two of the above combinations do not form a contrariety for “to do good to friends” is not the contrary of “to do harm to enemies” [$((((s \rightarrow u) / (s \leftrightarrow u)) \leftrightarrow ((\neg s \rightarrow \neg u) / (s \leftrightarrow u)))$ (1 1 1 1) and $((((s \rightarrow u) / (s \nleftrightarrow u)) \leftrightarrow ((\neg s \rightarrow \neg u) / (s \nleftrightarrow u)))$ (1 1 1 1)]; for both these actions are objects of choice and belong to the same character [the above formalization explains Aristotle's words: the formulas are logically equivalent under both the condition $(s \leftrightarrow u)$ (1 0 1 0) and its contrary $(s \nleftrightarrow u)$ (0 1 0 1)]. Nor is “to do harm to friends” the contrary of “to do good to enemies” [$((((\neg s \rightarrow u) / (s \leftrightarrow u)) \leftrightarrow ((s \rightarrow \neg u) / (s \leftrightarrow u)))$ (1 1 1 1) and $((((\neg s \rightarrow u) / (s \nleftrightarrow u)) \leftrightarrow ((s \rightarrow \neg u) / (s \nleftrightarrow u)))$ (1 1 1 1)]; for both these actions are objects of avoidance and belong to the same character [as in the previous case, they are logically equivalent under the condition $(s \leftrightarrow u)$ (1 0 1 0) and also under their contrary $(s \nleftrightarrow u)$ (0 1 0 1)], and one object of avoidance is not generally regarded as the contrary of another object of avoidance, unless the one is used to denote excess and the other defect; for excess is generally regarded as an object of avoidance, and so likewise also is defect. But all the other four combinations form a contrariety; for “to do good to friends” is the contrary of “to do harm to friends” [$(((s \rightarrow u) / u) \nleftrightarrow ((s \rightarrow \neg u) / \neg u))$ (1 1 1 1)], for they

> proceed from contrary characters, and one is an object of choice and the other of avoidance. Similarly, also, with the other combinations; for in each pair, one is an object of choice, the other of avoidance; one always belongs to a good character, the other to a bad. It is obvious, therefore, from what has been said that the same thing has in fact more than one contrary. For "to do good to friends" has as its contrary both "to do good to enemies" [$(((s \rightarrow u) / \neg s) \nleftrightarrow ((\neg s \rightarrow u) / s))$ (1 1 1 1)] and "to do harm to friends." In like manner, if we examine them in the same way, it will be apparent that the contraries of each of the others are two in number [$(((\neg s \rightarrow \neg u) / s) \nleftrightarrow ((s \rightarrow \neg u) / \neg s))$ (1 1 1 1) and $(((\neg s \rightarrow \neg u) / \neg u) \nleftrightarrow ((\neg s \rightarrow u) / u))$ (1 1 1 1)]. (*Topics* 113a1-19: Forster translation, 1960).

When constructing contradictions, we can assume in each conditional its consequent or the denial of its antecedent. That explains why "the same thing had more than one contrary." The text is particularly intriguing because it incorporates the six possible conditions considered about the squares of Figure 1 ($s \leftrightarrow u$, $s \nleftrightarrow u$, $\neg u$, u, $\neg s$, and s).

§ 6. Conclusions

We can condition the four propositions of the Aristotelian diagrams to depend on different assumptions and then analyse how they behave toward each other. Implementing this idea facilitates a homogeneous reading of some texts relative to the Aristotelian doctrine of opposition. It reveals the existence of distinctions to be faced outside predicate logic. We have advanced in Williamson's line of work: modern truth tables and propositional calculus can contribute to a better understanding of Aristotelian logical analysis.

The possibilities offered by the relationships between sentences, under the principle of conditioning, deserve to be carefully explored. Numerous passages of the *Organon* rely on barely expressed diagrams of opposition. And it remains defiant to adapt the idea to categorical propositions or even Aristotelian modal logic. (Given two conditioned formulas, the fact that one of them is or is not necessary, possible, admissible, or impossible affects the modal qualification of the other).

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Some notes on the Aristotelian doctrine of opposition and the propositional calculus

In this paper, we develop some of Williamson's ideas about the contribution of propositional calculus to a better understanding of Aristotelian logic. Specifically, the use he makes of truth tables in analysing the structure of the traditional square of opposition is enhanced by a simple technique: The relations of dependence between different propositions allow us to construct “conditioned truth tables.” Taking advantage of this possibility, we propose a new interpretation of several passages of the Organon related to opposition.

Keywords: Aristotle · Square of opposition · Ancient logic · Conditioned truth tables.

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HISTORIA DEL ARTÍCULO | ARTICLE HISTORY
Received: *30–Oct–2023*; Accepted: 15–*Nov–2023*; Published Online: *30–Dec–2023*

COMO CITAR ESTE ARTÍCULO | HOW TO CITE THIS ARTICLE
MATÍA CUBILLO, G.Ó. (2023). «Some notes on the Aristotelian doctrine of opposition and the propositional calculus». *Disputatio. Philosophical Research Bulletin* 12, no. 26: pp. 53–70.