On Bunge's axiomatizations of "partial truth" [1]

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1. Introduction

The concern about the nature of truth was a constant throughout Mario Bunge's philosophical career. One of the most notable theses he defended regarding this concept was that it comes in degrees, in contrast to the usual assumption that there are only two truth-values, an assumption which according to him is fully adequate only for the formal disciplines of logic and pure mathematics. Thus, we read in Bunge (1983, p. 256):

> Scientific research yields, at least with reference to the world of facts, partial truths rather than complete and accordingly final truths […] the scientific method, in contrast to the hit-or-miss procedure of common sense and unbridled speculation, is self-correcting: it can help recognize errors and obtain higher order approximations, i. e. truer answers.

And later on, in Bunge (2010, pp. 281-282):

> Asking how accurate a proposition is presupposes that there are truth values other than 0 and 1. This is a standard assumption in applied mathematics, factual science, and technology. Indeed, in all these fields it is taken for granted that the best one may ordinarily come up with is a good approximation to the truth – one that may eventually be perfected. […] In other words, one assumes, usually tacitly, that there is a truth

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valuation function V from some set P of propositions to a numerical interval […] Our problem is to come up with a plausible system of conditions (postulates) defining V.

This last passage expresses Bunge's desideratum of formulating an axiomatic theory that would formally elucidate notion of "degree of truth" – a notion which, as both cited passages show, is meant to be motivated by the fallible and progressive nature of scientific representation and inquiry.

He made various attempts at formulating such a theory. Bunge (1963) was, to the best of my knowledge, the first version. It was criticized by Ackermann (1964), with a reply in Bunge (1968), and by Miller (1977). The theory was later revised in Bunge (1974) and further in Bunge (1983), both of which versions were discussed and compared in Quintanilla (1985). Bunge's last attempts at finding a satisfactory set of axioms appear to have been Bunge (2010) and Bunge (2012). More recently this project has been taken up by Gustavo E. Romero in Romero (2017) and Romero (2018) and has received an interesting discussion in Marquis (2019).

Whatever may be said of the philosophical underpinnings of Bunge's theories, they all present some awkward formal problems. The purpose of this note is to take a look at these successive attempts of finding a theory of partial truth. I will briefly point out the formal defects of the earlier theories (only those which were not identified by the aforementioned critics), and I will prove that the latest formulations by Bunge and Romero are still not adequate – the most recent one being, in fact, inconsistent.

2. The 1963 system.

Bunge's first axiomatization of the notion of partial truth was presented in Bunge (1963, p. 118). The axioms we find there are the following[2]:

Axiom 1: $-1 \leq V(p) \leq 1$.

Axiom 2: $V(\sim p) = -V(p)$.

[2] I will adhere to Bunge's notation and use p, q, r ... throughout as variables for sentences of any complexity, not only atomic ones. It should also be noted that for Bunge the assignment of a truth-degree to a statement is always relative to a body of evidence and can change if that evidence changes. (Thanks to G. E. Romero for pointing this out to me). This is a philosophically interesting and controversial point, but it does not play an important role in the subsequent discussion: all assignments of truth-degrees considered here can be understood as properly relativized to some set of evidence.

Axiom 3: (i) If $V(p)$ and $V(q)$ are not both 0, then:

$$V(p \wedge q) = V(p) + V(q) - \left(\frac{V(p) \cdot V(q)}{V^2(p) + V^2(q)} \cdot \left(V(p) + V(q) - 2\delta_{V(p),-V(q)}\right)\right),$$

Where δ is the Kronecker delta, so $\delta_{V(p),-V(q)} = 1$ if $V(p) = -V(q)$, and 0 otherwise.

(ii) If $V(p) = V(q) = 0$, then $V(p \wedge q) = 0$.

An initial difficulty is how to interpret the notation "$V^2(p)$", which is not standard and not is explained in the book. I had the opportunity to consult Bunge on the matter in 2018 and he pointed out to me (in personal correspondence) that this was an erratum and that the superscripts ought to be omitted. But this correction doesn't work, for consider the contradictory formula $p \wedge {\sim}p$ for $V(p) \neq 0.$ If we then apply axioms 2 and 3 (omitting the superscripts), we have:

$$V(p \wedge {\sim}p) = V(p) - V(p) - \left(\frac{V(p) \cdot -V(p)}{V(p) - V(p)} \cdot \left(V(p) - V(p) - 2\delta_{V(p),-V(\sim p)}\right)\right)$$

But this requires us to divide by 0, so $V(p \wedge {\sim}p)$ is not defined. And given the intuitive interpretation of the function V, the truth-degree of $p \wedge {\sim}p$ ought to be "completely false", that is, -1. Let's consider a more natural reading of "$V^2(p)$" and treat it as "$V(p)^2$". Then clause (i) of axiom 3 should read thus:

$$V(p \wedge q) = V(p) + V(q) - \left(\frac{V(p) \cdot V(q)}{V(p)^2 + V(q)^2} \cdot \left(V(p) + V(q) - 2\delta_{V(p),-V(q)}\right)\right)$$

However, this amendment is still defective. The reason is that the function V can now assign different values to formulas which are mutually deducible (i. e. logically equivalent). Let $V(p) = 0.5$. By the propositional calculus of classical logic, we know that p and $p \wedge {\sim}(p \wedge {\sim}p)$ are equivalent. One would then expect Bunge's axioms to yield $V(p) = V\big(p \wedge {\sim}(p \wedge {\sim}p)\big)$ for any degree of truth assigned to p (at least in the book he says nothing to suggest that this shouldn't necessarily be so). Applying axioms 2 and 3 (with the mentioned correction) we can show, first, that $V\big({\sim}(p \wedge {\sim}p)\big) = 1$ (which is evidently a natural result). But if we then apply axiom 3 to compute the truth-degree of $p \wedge {\sim}(p \wedge {\sim}p)$ we get that $V\big(p \wedge {\sim}(p \wedge {\sim}p)\big) = 0.9$. So $V(p) \neq V\big(p \wedge {\sim}(p \wedge {\sim}p)\big)$.

As we will see below, this is a problem that will re-emerge in the systems of 1983 and 2010.

3. The 1974 system

In Bunge (1974b, p. 110) we find a simplification of the 1963 axioms. They are undoubtedly more perspicuous, but nonetheless unsatisfactory:

Axiom 1: $V(p \vee q) = V(p) + V(q) - V(p \wedge q)$.

Axiom 2: If p is a contradiction, $V(p) = 0$.

Axiom 3: If p is a tautology, $V(p) = 1$.

Although Bunge has later repudiated all identification of "partial truth" with probability, the similarity between these and Kolmogorov's axioms for probability functions over a propositional language is striking (Quintanilla 1985). But this resemblance has unwelcome consequences.

By axiom 1 we have that $V(p \wedge \sim p) + V(p \vee \sim p) = V(p) + V(\sim p)$ which, by axioms 2 and 3, yields that $V(\sim p) = 1 - V(p)$. Now consider the statement "the beam x is 1 meter long". Suppose it is completely true. Then its truth-degree should be 1. It would then be quite natural to say that the statement "the beam x is 99 centimetres long" has a truth-degree of 0.99, that is, it is "almost true". But now take the negation of this second statement: "the beam x is *not* 99 centimetres long". Intuitively, this statement is true *simpliciter*, yet by the axioms its truth-degree turns out to be $1 - 0.99 = 0.1$. In other words, the 1974 theory has the implausible consequence that a statement which seems to be completely true is actually almost completely false[3].

4. The 1983 system.

The above difficulty led Bunge to revise the theory once more in volume VI of his *Treatise on Basic Philosophy* (Bunge 1983, pp. 273-274). There, negation receives a more delicate treatment, and the connectives of conjunction and disjunction begin to resemble their counterparts in many systems of fuzzy logic (Priest 2008, chapter 11):

Axiom 1: (i) If p is not the negation of a statement[4] q, then:

[3] On this issue, see Quintanilla (1985, p. 134).

[4] Bunge uses the word "proposition", but since he is clearly referring to linguistic entities, I will adapt his terminology.

$V(\sim p) = 0$ if $V(p) = 1$, and $V(\sim p) = 1$ if $V(p) < 1$.

(ii) If p is the negation of a statement q which is not in turn the negation of another statement, then $V(\sim p) = V(q)$.

Axiom 2: $V(p \land q) = min\{V(p), V(q)\}$.

Axiom 3: $V(p \lor q) = max\{V(p), V(q)\}$.

It is clear that axiom 1 avoids the problem presented by negation in the 1974 system. However, this axiomatization has the same kind of problem that the original 1963 system: the function V can assign discrepant values to logically equivalent statements. To give an example, consider $p \land q$ and $\sim(\sim p \lor \sim q)$, which are equivalent in classical logic (De Morgan's Laws), and let $V(p) = 0.8$, $V(q) = 0.9$. By axiom 2 we have $V(p \land q) = \min\{0.8, 0.9\} = 0.8$. On the other hand, axiom 1 entails that $V(\sim p) = V(\sim q) = 1$, and applying axiom 3 we get that $V(\sim p \lor \sim q) = \max\{1,1\} = 1$. Now, by axiom 1, $(\sim(\sim p \lor \sim q)) = 0$. So $V(p \land q) \neq V(\sim(\sim p \lor \sim q))$. In fact, with the same assignment of truth-degrees to p and q we can show that $V(p \lor q) \neq V(\sim(\sim p \land \sim q))$.

So, this system of partial truth does not preserve De Morgan's laws. This is problematic for several reasons. First, De Morgan's laws are central to logic. And not only to classical logic, but also to much weaker "many valued" logics like LP, K3, B3 (Bochvar's three-valued logic) and First Degree Entailment (see Priest 2008, chapters 7 and 8). Second, as a consequence, the biconditional behaves strangely: from the axioms and the usual definition of the material biconditional one can show that $V((p \land q) \leftrightarrow \sim(\sim p \lor \sim q)) = 1$, but we have seen that So $V(p \land q) \neq V(\sim(\sim p \lor \sim q))$; so, the biconditional no longer represents "sameness of truth value", as it usually does. Third, the fact that $V(p \land q) \neq V(\sim(\sim p \lor \sim q))$ goes against Bunge's own conception of the content of a statement. At several points he suggests that logically equivalent statements express the same proposition or have the same content (e. g. Bunge 1974a, pp. 18, 121, 129, corollary 4.7); at a point he even says that $P \lor Q$ and $\sim(\sim P \land\sim Q)$ are the *same* proposition (*ibid.,* p. 126). How could it be, then, that his theory of partial truth can assign different values to statements which express the same proposition? This issue is actually aggravated by the fact that Bunge's 1983 axioms are explicitly formulated in terms of propositions.

5. The 2010 program.

After these successive failed attempts at axiomatizing the theory of partial truth, Bunge did not try to patch it up again. In Bunge (2010) and Bunge (2012) he did, however, lay down a series of desiderata for future developments of the theory. These desiderata are, so to speak, guiding principles for the formulation of an adequate theory of partial truth:

D1. If p is a quantitative statement[5] that has been found to be true within the relative error ε, then $V(p) = 1 - \varepsilon$.

D2. If p is not the negation of another statement, then $V(\sim p) = 0$ if $V(p) = 1$, and $V(p) = 1$ if $V(p) < 1$. If p is the negation of q, then $V(\sim p) = V(q)$.

D3. For any statements p and q, if $p \Leftrightarrow q$ [sic], then $V(p) = V(q)$.

D4. If p is not the negation of q, then $V(p \wedge q) = [V(p) + V(q)]/2$. If p is the negation of q, then $V(p \wedge q) = V(\sim q \wedge q) = 0$.

D5. For any statements p and q, $V(p \vee q) = \max\{V(p), V(q)\}$.

Bunge is quite explicit about the purpose of these conditions: they are meant as guiding principles for future developments of the theory, not as definitive axioms. In fact, he acknowledges that there may be reasons to reject or modify some of them:

> [The] project consists in figuring out a consistent set of postulates satisfying the above desiderata, some of which should occur as axioms, others as theorems, and perhaps still others in altered form or even not at all. (Bunge 2010, pp. 284-285).

Nonetheless, Gustavo E. Romero has taken all of these desiderata as axioms in his recent formulations of the theory of partial truth in Romero (2017) and Romero (2018). I will now show that this set of desiderata, and hence Romero's axiomatization, is inconsistent.

First of all, the meaning of D3 has to be clarified, since it involves the symbol "$\Leftrightarrow$" which is ambiguous. If it is interpreted as the object-language material biconditional, then the natural reading of D3 would be:

*D3**. If $V(p \leftrightarrow q) = 1$, then $V(p) = V(q)$.

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[5] See note 3.

If "⇔" is understood as the (metalinguistic) relation of logical equivalence, then the obvious way to read D3 would be this:

*D3***. If p and q are logically equivalent[6], then $V(p) = V(q)$ for any assignment of degrees of truth to the atomic statements of the language.

The problem is that neither of these readings of D3 is consistent with the rest of *desiderata.* Let's consider D3* first and let $(p) = 0.2$, $V(q) = 0.6$. Then $V(p \leftrightarrow q) = V\big((\sim p \vee q) \wedge (\sim q \vee p)\big)$ which by D4 is equal to $[V(\sim p \vee q) + V(\sim q \vee p)]/2$. Now, applying D5, this is $[\max\{V(\sim p), V(q)\} + \max\{V(\sim q), V(p)\}]/2$, which by D2 is equal to $[\max\{1, 0.6\} + \max\{1, 0.2\}]/2 = 1$. Hence, we have that $V(p \leftrightarrow q) = 1$. But $V(p) \neq V(q)$, which contradicts D3*.

Let's take D3** now. This yields at least two inconsistencies. First, there are assignments of truth-degrees to atomic statements such that $V(p \wedge q) \neq V\big(\sim(\sim p \vee \sim q)\big)$, which contradicts D3**. To give an example, let $V(p) = 0.9$, $V(q) = 1$. Then, by D4, $V(p \wedge q) = [V(p) + V(q)]/2 = 0.95$. Now, applying D2: $V(\sim p) = 1$ and $V(\sim q) = 0$. Therefore, by D5, $(\sim p \vee \sim q) = \max\{1{,}0\} = 1$. And applying D2 again: $V\big(\sim(\sim p \vee \sim q)\big) = 0$. So $V(p \wedge q) \neq V\big(\sim(\sim p \vee \sim q)\big)$. (Notice that in the 1983 system something very similar happened, but there we didn't have a principle like D3**, at least not explicitly. What in that system was merely an awkward feature now becomes an outright inconsistency).

Second, and more disconcertingly, there are assignments of truth-degrees to atomic statements such that $V\big((p \wedge q) \wedge r\big) \neq V\big(p \wedge (q \wedge r)\big)$. For let $V(p) = 1, V(q) = 0.5, V(r) = 0$. Then, by two successive applications of D4, $V\big((p \wedge q) \wedge r\big) = [V(p \wedge q) + V(r)]/2 = [((1 + 0.5)/2) + 0]/2 = 0.375.$ But, on the other hand, again by D4: $V\big(p \wedge (q \wedge r)\big) = [V(p) + V(q \wedge r)]/2 = [1 + ((0.5 + 0)/2)]/2 = 0.625$. So we have that $V\big((p \wedge q) \wedge r\big) \neq V\big(p \wedge (q \wedge r)\big)$, again contradicting D3**.

[6] This of course is also ambiguous since there are many different notions of logical consequence or entailment. But Bunge almost never considers a logic other than classical logic, and in this case gives no hint that a non-classical logic should be presupposed. This is also why we can disregard the distinction between the syntactic and the semantic notion of logical consequence/equivalence, since in classical propositional logic they coincide. However, he says that "in factual science the most useful concept of entailment is the syntactic not the semantic one" (Bunge 1974b:178).

This means that Romero's recent axiomatization in Romero (2017) and Romero (2018) is inconsistent and that Bunge's project of finding an adequate theory of partial truth remains open. For those interested in pursuing it, two options suggest themselves. One is to bite the bullet and accept the awkward features of the systems prior to 2010: loss of De Morgan's Laws (and, in general, possible discrepancies of truth-degree between logically equivalent statements), and strange behaviour of biconditional and negation. (Of course, whoever wants to go down that road will have to explain why such basic principles of logic as De Morgan's Laws should not be preserved). The other option would be to abandon Bunge's axiomatizations altogether and try to formalize the notion of partial truth using a system of fuzzy logic like Łukasiewicz's continuum valued logic (Priest 2008, p. 227). Both options presuppose, of course, that one wishes to continue thinking that the notion of "degrees of truth" is a coherent one. But that is a discussion which I leave for another occasion.

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Title: On Bunge's axiomatizations of "partial truth"

Abstract: One of the most notable theses Bunge defended regarding the concept of truth in empirical sciences was that it comes in degrees, in contrast to the usual assumption that there are only two truth-values, an assumption which according to him is fully adequate only for the formal disciplines of logic and pure mathematics. This paper discusses Bunge's successive attempts at formulating an axiomatic theory of partial truth. I will briefly point out the formal defects of his earlier theories and I will prove that the latest formulations of his theory (made by himself and G. E. Romero) are still not adequate – the most recent one being, in fact, inconsistent.

Keywords: Truth, Degrees of Truth, Empirical Sciences, Formal Sciences.

Título: Sobre las axiomatizaciones de "verdad parcial" de Bunge

Resumen: Una de las tesis más notables que Bunge defendía acerca del concepto de la verdad en las ciencias empíricas era que se da en grados, contrariamente a la suposición usual de que haya sólo dos valores de verdad; una suposición que es totalmente adecuada sólo para las disciplinas formales de la lógica y matemática pura. Este trabajo discute los sucesivos intentos de Bunge de formular una teoría axiomática de verdad parcial. Señalaré brevemente los defectos formales de sus teorías más tempranas y demostraré que las formulaciones últimas de su teoría (elaboradas por él mismo y G. E. Romero) siguen siendo inadecuadas – la más reciente siendo, de hecho, inconsistente.

Palabras claves: Verdad, Grados de verdad, Ciencias empíricas, Ciencias formales.

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