Mathematics, Intuition, and Language in the first *Critique* and the *Tractatus*

Paolo Degiorgi

> The question whether intuition [*Anschauung*] is needed for the solution of mathematical problems must be given the answer that in this case language itself provides the necessary intuition.
>
> The process of *calculating* serves to bring about that intuition.
>
> L. Wittgenstein, *Tractatus Logico–Philosophicus*, 6.233–6.2331[1]

THIS PASSAGE FROM WITTGENSTEIN'S *Tractatus* can be read as a reference to Kant's notorious claim that mathematics relies on *Anschauung*, usually translated into English as "intuition". There is good evidence for doing so. Wittgenstein had been familiar with this view since his early years in Cambridge; allusions to Kant's philosophy of mathematics can be found in his *Notebooks* starting from October 1914[2]. He is known to have read the *Critique of Pure Reason* with his friend Ludwig Hänsel towards the end of 1918, just a few months after completing the manuscript of the *Tractatus* (Monk 1991, p. 158). In the work, Kant is explicitly mentioned only at 6.36111; however, a profound awareness of and concern with Kantian problematics permeates the book[3]. It should also be noted that later work by the Austrian philosopher explicitly addresses the Kantian view about intuition and

[1] Henceforth the quotes from the *Tractatus* (TLP) will be marked simply by section number. I will be mostly quoting from the Pears/McGuinness translation as in (Wittgenstein 1974).

[2] See the entry from 19.10.1914 in (Wittgenstein 1979, p. 15).

[3] Several critics have noted the remarkable correspondence between the five main topics treated in sections 6.1 to 6.5 of the Tractatus (namely logic, mathematics, physical laws, ethics, and philosophy) and those in which Kant had located the sources of *a priori* knowledge.

Paolo Degiorgi (✉)
Boston University, US
e–mail: degiorgi@bu.edu

Disputatio. Philosophical Research Bulletin
Vol. 11, No. 23, December 2022, pp. 137-164
ISSN: 2254–0601 | [EN] | **ARTICLE**

mathematics[4].

The project of this paper will be to dispel certain interpretations that the passage might seem to invite. Consider A. W. Moore's chart illustrating a "rough" isomorphism "between, on the one hand, Kant's contrast between thoughts which have content and thoughts which lack content and, on the other hand, Wittgenstein's contrast between propositions which have sense and some nonsensical pseudo–propositions": (Moore 2013, p. 251)

	Left–Hand Side	*Right–Hand Side*
Kant	Thoughts with content	Thoughts without content
Wittgenstein	Propositions with sense	Certain nonsensical pseudo–propositions

As he goes on to spell out the content of the four boxes, he locates "the practice of mathematics" in Wittgenstein's Right–Hand Side[5], while he does not place mathematical cognition in Kant's Right–Hand Side (Moore 2013, p. 252). This reading follows readily from Kant's aforementioned claim and Moore's earlier characterization of Kantian "content" in terms of being about "objects of sensible intuition" (Moore 2013, p. 249)[6]. Thus, by Moore's lights, TLP 6.233 amounts to an implicit acknowledgment of the difference between Wittgenstein's and Kant's placements of mathematics in the chart. One might then be tempted to conclude that the chart captures what distinguishes the conceptions of mathematics at play in the *Tractatus* and the *Critique.*

Another author whose work has a say on the matter is Sören Stenlund. His sophisticated and intriguing historical analysis also places Kant's and Wittgenstein's conceptions of mathematics on different sides of the equation. His work, quite correctly, distinguishes two major conceptions at play in the history of western mathematics. The first, which he calls the "ontological" conception, is characteristic of mathematical thought in ancient Greece, paradigmatically in Euclid's *Elements*, and has continued to exert its influence ever since. Its defining characteristic is the immediate connection of mathematics to empirical applications, exemplified by the mandate that

4 See, for instance (Wittgenstein 1984, p. 129).

5 See p. 253. This is not surprising, as Wittgenstein refers to the equations of mathematics as "pseudo–propositions".

6 Moore quotes A51/B75 from Kant's *Critique of Pure Reason*.

mathematical concepts must prove their meaningfulness by having the potential to be exhibited in the physical world (Stenlund 1990, pp. 148–49)[7]. Ontological mathematics can be summed up as the science of magnitude. According to Stenlund, this was the predominant mathematical paradigm until at least well into the 18th century, and its conceptions are basic to Kant's *Critique* (Stenlund 2014, p. 39).

Stenlund labels the competing paradigm the "symbolic" conception. In this tradition, physical applicability stops being essential to mathematical notions as such. Mathematics is thought to be an essentially symbolic activity, whose central notions are variables, expressions, and formulae, not thought of as mere signs, but rather as elements of a broader operational system. Stenlund provides a variety of examples to elucidate this distinction; for instance, he contrasts the Greek idea of number, which is always the number *of things*, to the notion of natural number employed by contemporary mathematicians, which can be represented by the indeterminate number variable n as it appears in a proof by mathematical induction (Stenlund 2014, pp. 12–14). Early thinkers in the analytic tradition are, for Stenlund, the principal philosophical witnesses of the emergence of the symbolic conception; however, he singles out Wittgenstein in his continuous effort to assign a prominent place to symbolic mathematics in his thought[8]. It should be clear, from this exceedingly brief summary of the distinction, that Stenlund might read TLP 6.233 as further evidence for the fact that Kant and Wittgenstein were concerned with accounting for different mathematical traditions.

While I do not deem the former conclusion to be incorrect, in this paper I will take a route different from both Moore's and Stenlund's. I shall propose readings of the *Critique* and the *Tractatus* that will allow me to bring out some substantial structural similarities and points of agreement between the conceptions of mathematics at play, showing that the authors were concerned with similar issues about the nature of the subject and had similar ideas of what

7 What follows is an extremely succinct summary of (Stenlund 2013) and (Stenlund 2014). My intent here is not to provide a complete account of Stenlund's historical analysis, but rather to illustrate how various exegetical routes have led interpreters to different characterizations of the contrast between Kant's and Wittgenstein's conceptions of mathematics.

8 See (Stenlund 2013, p. 14): "Wittgenstein's conception of mathematics, already from the beginning in the *Tractatus*, has much in common with what has been called symbolic mathematics. I think that this is true of the early and middle as well as the late Wittgenstein". See also p. 21.

kind of account mathematics calls for. Doing so will involve a broader understanding of both texts. I will ground my account on the fundamental structural affinity I discern between the Tractarian notion of *Satz* (translated as proposition or sentence) and the Kantian one of *Erkenntnis* (cognition, or knowledge). I shall argue that both terms are fruitfully understood by considering their paradigmatic employment: empirical cognition for Kant and the significant proposition for Wittgenstein. For both thinkers, mathematics is best understood through an analysis of these notions in terms of *form* and *matter*, aimed at elucidating the relationship between the propositions or cognitions employed in mathematics and those that bear the aforementioned paradigmatic form. I will call this analysis "hylomorphic," and shall soon clarify the reasons behind my employment of the Aristotelian term. The resulting hylomorphic account of mathematics that I shall discern in both Kant and Wittgenstein creates a distinction transversal to Stenlund's ontological–symbolic one and questions the framework behind Moore's equation. My intent is to deny the legitimacy of any exegetical strategy that appraises the differences between the two views of mathematics without first acknowledging their substantial structural affinities. Only after doing so shall I be positioned to briefly hint at how my framework could help us assess where the two accounts diverge; this should help us evaluate the significance of TLP 6.233.

§ 1. Kant's hylomorphism[9]. Mathematics in the first *Critique*

Kant famously claimed that mathematical judgments are synthetic *a priori*. This claim, by itself, addresses two classical concerns of any philosophy of mathematics. Because a mathematical cognition is *a priori*, its justification is independent of any particular experience; additionally, its being synthetic underlies the fact that mathematics is an *ampliative* science, capable of extending our knowledge[10]. In the introduction to the *Critique* Kant argues that the judgment 5+7=12 is not analytic because it cannot be obtained from the mere concepts of 5 and 7 and the principle of non–contradiction (Kant 2007b, B15); in order to go beyond these concepts, one must seek the assistance of

[9] The notion of hylomorphism I am concerned with in the present paper belongs to the historical tradition I am attempting to chart; my intent is not to engage with discussions of hylomorphism in contemporary metaphysics.

[10] For an explicit declaration of these motivations, see "Letter to Johann Schultz," Nov. 25 1788, in (Kant 2007a, p. 283).

intuition. Understanding this claim will be our task for this section[11].

The fact that "mathematics relies on intuition" might be taken to mean, as in Stenlund's ontological conception, that mathematical concepts must be applicable to experience in order to be meaningful. While this is certainly part of what Kant had in mind, I believe that he has a more nuanced story to tell. The intuitions involved in representing a number or carrying out a calculation could not possibly be merely empirical, otherwise the corresponding cognitions would not be *a priori*. Indeed, Kant maintains that mathematical concepts and judgments must be exhibited in *pure* intuition (B147, A711/B739)[12]. At this point, one might wonder whether the intuition on which mathematics relies is itself also *empirical*, and if not, why Kant wrote that mathematics relies on intuition *simpliciter,* and not simply on *pure* intuition. To address this, it is crucial to develop a proper understanding of the notion of pure intuition. At the beginning of the *Transcendental Aesthetic,* Kant introduces several of his central notions by drawing *form/matter* distinctions. For example, he calls the form of a sensible intuition *pure intuition,* (A20/B34), which suggests that we can think of *empirical* intuition as its matter, to which sensation belongs. I suggest that we read this distinction as part of a broader hylomorphic analysis of human cognition[13]. We can read the renowned distinction between Kantian *concepts* and *intuitions* as a form/matter analysis of cognition [*Erkenntnis*][14], in which we abstract concepts by attending to cognitions insofar as they are the product of the spontaneous faculty of the understanding, and, similarly, we

[11] Kant's claim is not solely concerned with the statement 5+7=12, nor just with the judgments of arithmetic, nor just with all mathematical propositions. It concerns all mathematical knowledge. The necessity of intuition is invoked in every mathematical example from the *Critique*: representing a number (B179), a triangle (B180), or a line (B203/A162), establishing geometrical principles or axioms (B16, B204/A163), proving geometrical results (A716/B744), manipulating algebraic expressions (A717/B745), and even representing irrational numbers – see his "Letter to August Wilhelm Rehberg" from Sept 1790, in (Kant 2007a, p. 356).

[12] For a more comprehensive discussion of this claim, see (Young 1992, p. 164).

[13] It is helpful to keep in mind Kant's discussion of matter and form in the Appendix to the Transcendental Analytic, A266–8/B322–324. It is crucial that we understand the hylomorphic analysis to be a species of transcendental reflection; the terms "matter" and "form" are not to be understood as empirical determinations. This means, as I shall emphasize later, that the matter and the form of a hylomorphic whole are not to be understood as "composed" of two self–standingly intelligible components. In this sense, matter and form are not "parts" of the hylomorphic whole.

[14] See (Conant 2016) for a more extended argument for adopting this reading of the *Critique*.

attend to intuitions by considering the contribution of the receptive faculty of sensibility[15]. Concepts, in turn, can be considered in their mere form (as *pure* concepts), which is the subject of Pure General Logic[16], or in their relation to objects *a priori* (as *empirical* concepts), the subject of Transcendental Logic. This parallels the form/matter analysis of intuitions discussed above, as Kant remarks at A55/B80. We thus obtain a four–fold analysis of cognition, which can therefore be studied from four different perspectives. The one that concerns mathematics, as we have said, is that of pure intuition. In order to attend to a pure intuition, one has to "separate from the representation of a body that which the understanding thinks about it […], as well as that which belongs to sensation […]" (A20–1/B35); in other words, one has to first focus on the *matter* of cognition by disregarding its conceptual form, and then consider the spatio–temporal *form* of what is left, by abstracting from its empirical or sensible matter (color, taste, etc.).

Following Kant's employment of the Aristotelian terminology, I suggest that we think of the *form* and the *matter* of a representation (a cognition, an intuition, a concept, etc.) as being *logically inseparable*. This means that, for example, a cognition should be thought of as a *totum* whose unity is logically prior to its parts, namely its *form* and its *matter*; the two can be *ab*stracted from it, but not *ex*tracted; in other words, the analysis of the representation into these components is logically secondary to their synthesis. The two are *notionally* separable, but cannot be understood independently of each other, nor encountered in isolation[17]. The hylomorphism I have in mind is not best characterized merely as a conception about form and matter, but rather one about *unity*, supplying the vocabulary to talk about the internal articulation of organic wholes such as Kantian cognitions. In labeling Kant's analysis of cognition and the resulting conception of mathematics "hylomorphic," I thus primarily intend to emphasize the fact that the *unity* of cognition must be presupposed in any discussion of the nature of mathematical cognition and pure intuition.

Contrast this with Michael Potter's picture of the same relations, which I take to be similar to the one working in the background of Moore's table. Potter describes concepts as the components of knowledge "that relate to objects

[15] For a nice characterization of sensibility in these terms, see (Smyth 2014).

[16] See the introduction to the *Transcendental Logic*, esp. A50–1/B74–5, and A55–7/B79–82.

[17] Again, I am borrowing from (Conant 2016).

indirectly," and intuitions as those "that do so directly;" he then says that in order to get genuine knowledge, these two components have to be "combined," or "connected" (Potter 2002, p. 21). This might make it seem as if Kant sorted thoughts into two symmetrical boxes, as in Moore's table: those that have been "connected" to intuitions ('with content'), and those that have not ('without content'). Hence, we could have, for instance, pure mathematical thoughts that lack intuitive content, but nevertheless amount to knowledge. Stenlund's characterization of Kant's understanding of mathematics as "ontological" would not save him from a similar possibility: mathematical concepts might be thought of as self–standingly intelligible even admitting that their *meaningfulness* depends on empirical application. In my reading, Kant would deem such an account incapable of truly explaining the applicability of mathematics to all appearances. The connection between concepts and their empirical instantiations would be simply not strong enough: any further philosophical trick would at best indicate a logically secondary connection between mathematical concepts and their empirical instances. This is just another guise of the insurmountable impasse faced by any "platonist" account of mathematics. Fortunately, the question does not arise for Kant, as I hope to have made clear by stressing the internal relation between concepts and intuition. Kant's hylomorphism about cognition implies that when he spoke of mathematical concepts being *constructed* in intuition, he meant it in a strong sense: these concepts would not be intelligible outside the intuitive context of construction.

A similar neglect of Kant's hylomorphism – this time at the level of intuition – leads Potter to conclude that, for Kant, there are two kinds of intuition: "sensibility supplies us not only with empirical but with *pure* intuitions, i.e. intuitions deriving not from experience itself but from its structure" (Potter 2002, p. 40); he also calls the latter "intuitions that are in some sense spatial but yet *not empirical*" (Potter 2002, p. 25; my emphasis). To see why, exactly, I deem this setup misleading, let me first dwell on what the pure intuition in an empirical cognition looks like from the the hylomorphic perspective I have been proposing. Pure intuition does not differ from empirical intuition in its target; it is *not* an intuition of a special kind of object[18], nor of a concept. One can

[18] See also A291/B347: "The mere form of intuition, without substance, is itself not an object [*Gegenstand*], but the merely formal condition of one (as appearance), like pure space and pure time, which are to be sure something, as the form for intuiting, but are not in themselves objects that are intuited (*ens imaginarium*)." Even though talk of "mathematical objects" was not as widespread as it is today, Kant argues against the view that pure intuition has its own special "objects".

answer the questions "what is the pure intuition of the image of five an intuition *of*?" and "what is the pure intuition of the sum of five and seven an intuition *of*?" by saying: "of *these* five dots – insofar as they bear numerical structure" and "of *these* twelve fingers (grouped in this succession)." Nor does pure intuition, in the case of empirical cognition, denote a distinct act from its corresponding empirical intuition. An act of intuiting involves *both* an empirical *and* a pure aspect. What makes an intuition *pure* is the way in which we attend to what is intuited; to do so "we [take] into account only the action of constructing the concept, to which many determinations […] are entirely indifferent, and thus we [abstract] from these differences, which do not alter the concept" (A714/B742). To enjoy a pure intuition means to abstract from the empirical content of an appearance and consider solely its spatio–temporal form, i.e. the way in which it is constructed. Kant's hylomorphism about intuition entails that mathematics must depend on both pure *and* empirical intuition: the two come as a single package. Hence Kant is justified in saying that mathematics depends on intuition *simpliciter*.

Potter, in defense of his unstructured partition of intuitions into pure and empirical, might point out that Kant does speak of mathematical cognitions that are merely pure intuitions, lacking empirical content. The *Critique* in fact speaks of "mathematics in its pure use" (A166/B206), and of constructing a geometrical figure in imagination (A224/B272, A713/B741), as opposed to constructing it "on paper, in empirical intuition" (A713/B741). I argue that these passages pose no threat to my interpretation. In the chapter on the postulates of empirical thinking, from which some of these passages are taken, Kant stresses that the synthesis through which we construct a mathematical concept in imagination is the same as that through which we apprehend an appearance of that concept (A713/B741). He also writes that "a pure concept […] nevertheless belongs to experience, since its object can be encountered only in the latter" (A220/B268). Even showing the mere *possibility* of the existence of such an object relies on experience:

> It may look, to be sure, as if the possibility of a triangle could be cognized from its concept in itself (it is certainly independent of experience); for in fact we can give it an object entirely a priori, i.e., construct it. But since this is only the form of an object, it would still always remain only a product of the imagination, the possibility of whose object would still

> remain doubtful, as requiring something more, namely that such a figure be thought solely under those conditions on which all objects of experience rest (A223–4/B271).

The pure intuition of a triangle constructed in imagination is still the mere form of an object, an appearance without determinate empirical content. We can make better sense of Kant's view if we think of cognitions arising from experience as the *paradigmatic* instances of cognition. This means that, strictly speaking, the four–fold analysis of *Erkenntnis* I presented should be applied primarily to empirical cognition, and only derivatively to other instances. We can understand the non–paradigmatic forms of cognition, such as remembering an image or drawing a triangle in imagination, as parasitic on previous instances of full–blown empirical cognition, and hence as being analyzable only from some of the four perspectives of our hylomorphic analysis. We can only remember an image because we have in fact seen it at some point in the past, as the object of an empirical cognition. Similarly, we can construct a triangle without having to draw it by relying on the capacity we exercise both in seeing triangle–shaped objects and in drawing triangles on paper. It is crucial, therefore, that we treat Kant's peripheral remarks on non–empirical pure intuition not as signaling a distinction between species of intuition – pure and empirical – but rather, as addressing our sophisticated capacity to engage in *partial* exercises of our capacities, i.e. of disregarding essential components of cognitions, thereby generating new, "deficient" representations that are nevertheless parasitic on full–blown empirical cognitions. What is missing in Potter's account is the determinate logical *structure* to the universe of Kantian cognitions; in particular, he fails to recognize the presence of a paradigmatic sort of cognition. Once we appreciate this structure, we also come to see how, given Kant's framework, the task of giving an account of mathematical cognition amounts to describing its relationship to full–blown, empirical cognition; in other words, to spelling out the details of the specific partial exercise of our *Erkenntnisvermögen* required for mathematical representation. This is the *shape* he thinks an account of mathematics should take. In the remainder of this section, I will delve deeper into Kant's remarks on mathematics to give the reader a glimpse into how this is done.

Merely attending to pure intuition does not exhaust, for Kant, the skillset of a mathematician. Let us dwell on the respects in which representing a number differs from adding two numbers. As we saw, judging that 5+7=12 involves bringing together the concepts of five and seven constructed in intuition;

however, this does not merely mean bringing the images of the concepts together in pure intuition in a generic way; otherwise, we would not be able to distinguish the representation of 5+7 from that of 5–7. Adding five fingers to seven fingers means bringing the two images together *in succession.* The arithmetic operation of *addition* thus denotes the *form* of the act involved in the intuitive construction[19]: its matter consists of the representations of five and seven. In other words, even at the level of pure intuition, we can make further form/matter distinctions. Mathematical operations and constructions of higher complexity can in fact be analyzed in terms of increasingly finer hylomorphic distinctions. This is related to how Kant, in B14–16, mentions the *concept* of five, that of seven, that of "the sum of seven and five," but never talks of "the concept of summation" *simpliciter.* It is not the case that he thought that we cannot form concepts of arithmetical operations; rather, I believe that he thought that talking about "the concept of sum" while discussing the judgment 5+7=12 would be misleading as it would conflate the form with the matter of the representation. In this operation, the plus sign merely signifies the form of the synthesis through which the concepts at play (that of seven and that of five) are brought together, in intuition, to form that of twelve[20]. What is being represented is the possibility of bringing these two concepts together in a successive way, so that the image of seven is represented as being *added* to that of five.

To put the previous points in terms of actual mathematical practice, think of the difference between the intuitions corresponding to (*i*) counting up to twelve with fingers and (*ii*) adding seven fingers to five fingers. The *empirical* intuitive aspects of (*i*) and (*ii*) might be absolutely indistinguishable from each other. More is true: even if we consider some of the pure content of the intuition, such as the unfolding of the fingers, as a succession of homogenous units, it is not yet possible to distinguish the two. Doing so requires taking two different "hylomorphic" perspectives *within* the level of pure intuition. Part of what constitutes the form of the simpler representation (such as the succession of the first *five* fingers) should be perceived as a constituent of the matter of the more complicated one. What distinguishes the "lower level" operation from the "higher level" one is *not* the presence of an extra empirical feature; what

[19] See (Parsons 1983, p. 140) for further discussion of the structure of this act.

[20] For textual support, see Kant's letter to Schultz, Nov 1788, in (Kant 2007a, p. 284): "the sign "+" signifies the synthesis involved in getting a third number out of two numbers".

matters is the formal features of the intuition that one attends to, the way one (actively) frames the intuition.

To further corroborate the claim that Kant's concern with mathematics revolved around how mathematical form is displayed in the way given concepts or images are brought into synthesis, let us dwell on a passage from his correspondence with the mathematician Johann Schultz in 1788:

> I can arrive at a single determination of a magnitude = 8 by means of 3+5, or 12–4, or 2×4, or 2^3, namely 8. But my thought "3+5" did not include the thought "2×4." Just as little did it include the concept "8," which is equal in value to both of these. [...]
>
> Now assuming ["3+4=7"] were an analytic judgment, I would have to *think* exactly the same thing by "3+4" as by "7," and the judgment would only make me more clearly conscious of what I thought. But since 12–5=7 yields a number =7 that is actually the same number I thought when I was adding 3+4, it would follow according to the principle "things equal to the same thing are equal to each other," that when I think "3 and 4" I must at the same time be thinking "12 and 5". And this does not jibe with my consciousness (Kant 2007a, p. 284).

In this letter, Kant is trying to argue for the claim that mathematical judgments are synthetic because they rely on intuition. Given that sensibility and understanding are the only two stems of cognitions, this amounts to showing that a mathematical judgment cannot arise from concepts alone. Recall that, for Kant, human understanding is essentially *finite*, meaning that it cannot give rise to infinitely complex thoughts, although it can make representations of them[21]. We might then paraphrase the argument as follows: if the judgment 3+5=8 were analytic —thus dependent on the understanding alone— arguably all other basic arithmetic computations would be as well. In particular, in thinking the concept of eight, I would be thinking of *all* the possible ways of obtaining it through basic arithmetic operations (1+7, 2+6, 9–1, 10–2, etc.). There are infinitely many such judgments. It is crucial to notice that what makes these judgments different is that they have different *contents,* as some of them involve the concept "five," others contain the concept "six," etc. Hence, my finite understanding would be able to think infinitely many *distinct* concepts

[21] See B39/40: "Now one must, to be sure, think of every concept as a representation that is contained in an infinite set of different possible representations (as their common mark), which thus contains these *under itself*; but no concept, as such, can be thought as if it contained an infinite set of representations *within itself*." I am borrowing the idea of connecting infinity to the givenness of intuition from (Smyth 2015).

contained in a single concept. This is simply impossible for a finite understanding. We must conclude that 3+5=8 is a synthetic judgment.

In mentioning this passage, my intent is not to evaluate the argument nor to reconstruct it in excessive detail; rather, I want to draw the reader's attention to how, for Kant, the fact that a mathematical concept, such as a number or a sum, admits of multiple ways of being constructed or construed is a crucial feature of his account of mathematics, capable of grounding his most fundamental claim regarding the subject. Moreover, I believe that the language of hylomorphism provides us with a rigorous framework to understand the multiple construability of mathematical concepts, since distinct form/matter analyses need not, in general, undermine each other. As we turn to the *Tractatus*, we shall see that Wittgenstein is also concerned with the fact that numbers can be construed and constructed in multiple ways; what I aim to show is that this concern is related to the employment of a hylomorphic framework that is structurally identical to Kant's.

§ 2. Arithmetic in the *Tractatus*

The *Tractatus* famously characterized mathematics as a "method of logic" (6.234) that consists in employing equations (6.22); these express no thought and hence are pseudo–propositions (6.2, 6.21). Wittgenstein was certainly not in the business of reducing mathematics to logic, and he devoted two sections of the text exclusively to the former. The 6.0s discuss the notion of number, the 6.2s deal with arithmetic equations. Because I intend to highlight the affinities with Kant's discussion, I will not depart from these mathematically elementary examples, similarly to how I focused on the passages of the *Critique* that discuss representing and adding numbers. However, I do not intend to take a stance on how one should extend the Tractarian account to more sophisticated mathematics. I do believe that a firm understanding of the wider themes of the book is necessary in order to appreciate the two brief stretches dealing with mathematics. Hence, I will devote the beginning of the present section to filling out the basic infrastructure that is at play in the 6.0s and 6.2s, by discussing the sign/symbol distinction and the Tractarian notion of operation.

§ 2.1. The *Tractatus* on sign and symbol.

The distinction between sign [*Zeichen*] and symbol is at the heart of the

Tractarian view of language[22]. Wittgenstein describes the symbol as "any part of a proposition that characterizes its sense" (3.31), and the sign as "the part of the symbol perceptible by the senses" (3.32). He goes on to write that "In order to recognize the symbol in the sign we must consider the significant use" (3.326). I suggest that underlying these comments is a conception in which the basic linguistic unit, whose paradigmatic instance is a logically articulate proposition (3.141), has both a significant and a sensible dimension. The significant use of a proposition (its function, or purpose, 3.341) is that of *symbolizing*, that is, expressing a sense (3.3, 3.31), which asserts the existence or non–existence of a state of affairs. The components of a propositional symbol are also symbols; they are defined by their contribution to the sense of the proposition. Thus, the components of a proposition should be understood as being embedded in a logically prior whole[23]. Following the terminology employed in 3.31 ("An expression is the mark of a form and a content"), I suggest that the *Tractatus* operates within a hylomorphic conception of linguistic expressions: a symbol is nothing but the linguistic *form* of a proposition; a sign is its sensible *matter*[24]. The two are internally related, thus logically inseparable: a proposition is essentially both sign and symbol. A sign is essentially *significant*: it plays a role in language. To be a sign *is* to stand in a certain relation to a symbol. The notion of a *mere sign* is parasitic on that of a significant sign; similarly the idea of a mere symbol is parasitic on that of a sensibly perceptible one. The internal relation between sign and symbol is at the heart of Wittgenstein's concern with a logically (or mathematically) perspicuous notation (or *Begriffsschrift*, see 3.325), in which the sensible form of signs mirrors the logical or mathematical form of the symbols expressed. An example of this would be symbolizing the operation of negation by turning a propositional sign "p" upside down (obtaining a shape like "d'); this would mirror the fact that ~~p is the same as

22 My discussion of sign and symbol follows that of (Conant 2020) especially pp. 874–947, as well as (Conant 2002, pp. 398–405).

23 This is a rough gloss of the context principle expressed at 3.3. It is not meant to deny the converse logical dependence of the proposition on its components. For an in–depth discussion of how the *Tractatus* manages to stand by Frege's context principle without rejecting compositionality, properly understood, see (Bronzo 2011).

24 Saying that the symbol is the linguistic form of a proposition does not amount to equating it to the structure in which the components of the propositions have been connected; rather, form in the *Tractatus* is "the *possibility* of structure" (2.033). I take this to imply, for instance, that the possibility of expressing a sense by means of connecting the same components in different ways is internal to a propositional symbol.

p. When this agreement of forms is achieved, it is possible to inspect the logical form of a propositional symbol by merely attending to the sign. Later we shall see how this is relevant to mathematics as well.

Our analysis should be reminiscent of the form/matter distinctions we found in Kant. In both cases, I am using hylomorphism primarily as a way of conceiving the unity of cognitions and propositions respectively, and thus of their internal articulation. For Wittgenstein, part of the point of analyzing the proposition in terms of sign and symbol is to emphasize the primacy of the unity of the proposition. I believe that we can carry the analogy with Kant further[25]. Recall that Kant makes further form/matter specifications at the level of concepts and that of intuitions. Similarly, in interpreting the *Tractatus*, we can draw further distinctions at the level of the sign, its form being the *repeatable* or *significant sign* —which is what we attend to, for example, when we determine that two marks on paper are instances of one and the same sign— and its sensible matter being the *mere sign* or *mark* (ink on paper, a sound wave, etc. *qua* physical entities). At the level of the symbol, we can distinguish its *logical* form, or syntax, from its matter — that is, its meaning [*Bedeutung*], in the case of names, or its sense [*Sinn*], in the case of propositions (3.3). The logical form of the symbol is the level at which, according to Wittgenstein, logic and mathematics operate. The difference between logical form and the meaning of a sign is made explicit at 3.33: "In logical syntax, the meaning of a sign should never play a role. It must be possible to establish logical syntax without mentioning the *meaning* of a sign". In other words, when we attend to the logical form of a symbol, we abstract from what it signifies by considering only the logical role it plays in a sentence. This process of abstraction is described at 3.315 as the result of successively turning the constituents of a proposition into *variables*, until one has gotten rid of all signs that have "arbitrarily determined meanings". This passage will certainly remind one of Kant's description of how one comes to attend to pure intuition at the beginning of the *Transcendental Aesthetic*. Both processes begin with a full–blown cognition or proposition, respectively; both proceed by successively disregarding certain aspects and concentrating on specific formal features: pure intuition, for Kant, and logical

[25] Neither my account of Kantian cognitions, nor that of Tractarian linguistic units should be read as a definitive analysis in terms of form and matter. Depending on one's purpose, one could establish countless different form/matter specifications in either text; each would illuminate different features of Kant's or Wittgenstein's conceptions. The one I am proposing is intended to bring out the similarities between Wittgenstein's and Kant's accounts of mathematics.

syntax, for Wittgenstein. What is crucial here is that both authors can be fruitfully read as using the language of hylomorphism; my hope is to show that this analysis is essential to the task of understanding either of the two accounts of mathematics.

The author of the *Tractatus* recognizes that asserting *sinnvolle* propositions is not the only way in which language is used. To put it differently, not all propositional signs symbolize; some do not express a sense. This might seem to contradict what has just been said, namely that to be a sign is to stand in a certain relation to a symbol. This apparent tension will certainly remind the reader of the one stemming from Kant's talk of "pure" mathematical intuitions. However, we can make sense of how the two claims cohere if we recognize that (i) a non–symbolizing sign can still have a function within language; and that (ii) any such use of a sign is parasitic on the paradigmatic practice of using it in the context of a proposition with sense. Thus, a non–symbolizing sign is still internally related to a symbol, just as "imaginary" intuitions are internally related to empirical intuitions. For Kant, cognition arising from experience can be treated as paradigmatic for all human cognition; for Wittgenstein, *sinnvolle* propositions are the paradigm of all use of language. Similarly to how Wittgenstein called mathematical equations "pseudo–propositions," Kant could have called non–empirical pure intuitions "pseudo–cognitions".

The "6s" of the *Tractatus*, that is, TLP 6 to 6.54 discuss several cases of complete propositional signs that do not symbolize. We can gloss this final stretch of the text as follows: each sequence (6.1s, 6.2s, up to the 6.5s) deals with a class of propositional signs that are not significant propositions, but nevertheless serve a certain purpose. The topics of these sections are respectively logic, mathematics, natural laws, ethics, and philosophy. Strictly speaking, these propositional signs do not say anything, as they do not express a sense. For instance, even though the propositions of logic are senseless [*sinnlos*], we can use them to bring out the logical form of significant propositions (6.12), and thus get clearer about them; alternatively, we can use logical tautologies them to *guide* inferences, or as *records* of possible inferences involving significant propositions. Similarly, mathematical equations can be used as records of past calculations and guides for future ones[26]. In Moore's

[26] Here I am borrowing the idea from (Kremer 2022). This account helps us understand how also the linguistic *use* of non–symbolizing signs can be understood as parasitic on significant discourse (see 6.1264). Notice that Kremer does address the case of logical contradictions as well.

chart, each of these classes of propositions would fall into Wittgenstein's right–hand side ("certain nonsensical pseudo–propositions"). The problem with this subdivision is analogous to the one I alluded to speaking of Kant: in ignoring the paradigmatic role of *sinnvolle Sätze*, and dividing Tractarian propositions into an unstructured dichotomy, Moore misses the subtlety of Wittgenstein's account; the result is that he is forced to locate Wittgenstein's conception of mathematics in the opposite bucket to Kant's, omitting the structural affinities of the two accounts. On the contrary, we are now in a position to see how, for both Kant and Wittgenstein, an explanation of mathematics must, in the first place, spell out the details of the relation between mathematical cognitions/propositions and the paradigm they depend on.

We can first apply this requirement to the "senseless" propositions of logic, such as tautologies. The hylomorphic framework elucidates their difference with meaningful propositional signs: tautologies latter share their form, or logical syntax, with the significant propositions, but not their matter (their sense). We can thus think of tautologies as sentences in which all the "meaningful" constituents can be replaced by the appropriate, non–symbolizing *variables* (as described in 3.315) without changing their sense, bringing out the logical form of the proposition — for instance, "Socrates is Socrates" says nothing more than "*x* is *x*." To develop a similar understanding of mathematical pseudo–propositions, and thus also come to see how these differ from those of logic, we first have to delve into more Tractarianese.

§ 2.2 *Tractarian* operations[27] and numbers

In the *Tractatus,* an operation is the expression of a relation between an initial proposition and another that is obtained from it (5.22). It marks their difference in form (5.241). In other words, we apply operations to one or more propositions to obtain new ones. Examples include logical constants, such as conjunction and negation (5.2341), as well as the generalized Sheffer stroke "N". These should be contrasted with Russellian propositional functions, such as "*x* is tall"[28]. Operations, unlike functions, can be applied repeatedly to their

[27] See (Hylton 2005), (Floyd 2002, pp. 314–418), (Marion 1998), or (Frascolla 1994, chap. 1) especially pp. 1–7.

[28] Notice 5.25: "Operations and functions must not be confused with each other." See (Hylton 2005, pp. 141–47) for an extended discussion of the difference between operations and the Russellian and Fregean

own results (5.251); for example, it is possible to concatenate negation signs. More generally, operations can be composed with each other (as in the case of ~ (*p* v *q*)); this allows them to "undo" the result of other operations (5.253). For example, "~~*p*" is *the same* proposition as "*p*" (meaning that it is the same *symbol*, it has the same sense), or "~((~*p*)&(~q))" is the same proposition as "*p* v *q*". Hence it is vital that operation signs *do not* symbolize (4.0312)[29]: they do not stand for anything, like punctuation marks (5.4611). Rather, they characterize the logical form of a propositional symbol. This means that they are "dispensable": for any given operation, it is possible to develop a logical notation that does not use a sign to express it. This does not mean that such notation would have no systematic way of expressing the operation; it would simply lack a distinct sensible sign for it.

The foregoing clearly implies that operations pervade all language. They are the most basic way to obtain significant propositions from other significant propositions; in fact, the only way, according to TLP 6, which expresses the general form of a proposition in terms of the "N" operation. This implies that operations are internally related to the significant bits of language, and that they are not "primitive logical signs" (5.461). Likewise, operations are internally related to one another, as pointed out in 5.451. This means, for instance, that understanding negation involves knowing how to compose it with conjunction. Wittgenstein's reflections thus stress how operations constitute an essential part of the internal unity of language as a whole.

We are now ready to discuss numbers, defined at 6.02–6.03 as exponents of a generic operation. Wittgenstein explicitly refuses to define numbers in terms of sets/classes (6.031) or in any similar manner. This partly due to the fact that, like Kant, he deemed it misleading to think of numbers as "objects;" to see this, notice that if we write the proposition "there are six cats" in a perspicuous logico–mathematical notation, this will not involve writing any numeral sign; the numerosity of cats will be displayed by repeated use of the existential quantifier.[30] Hence numbers are "dispensable," like signs denoting operations.

notions of functions.

[29] This is because, for example, if the negation sign characterized the sense of a proposition, then ~~*p* and *p* would differ in sense and hence they would not be the *same* proposition.

[30] Wittgenstein's appreciation of how proper notation (sign) is an *essential* component of mathematical notions is part of what makes him in Stenlund's reading, committed to a "symbolic" conception of mathematics – see (Stenlund 2013, pp. 19–20). For us, this is a consequence of thinking of sign and symbol

Similarly to how, for Kant, the internal relation between understanding and sensibility allows us to talk of constructing numerical *concepts* in *intuition*, Wittgenstein's hylomorphic conception of the relationship of sign and symbol allowed him to talk of *propositional signs* (the matter of a proposition) as displaying a numerical *form*. I take it that his definition of number is meant to suggest that operations are essential to numberhood. In light of what we saw in the previous paragraph, this means that our grasp of numbers "is not a further skill over and above our grasp of language" (Potter 2002, p. 178). We can talk about numbers *because* we can form propositions by concatenating operations in succession. This latter practice is, in turn, itself grounded in our understanding numerical succession. This suggests that numbers should be conceived as integral components of significant propositions, understood as wholes whose unity is prior to the parts. Hence Wittgenstein's "definition" should not be understood in the mathematical sense of the term[31], nor as a "reduction" of the notion of number to that of operation; rather, it is an attempt to individuate the role of numbers within language, as well as to highlight the intimate connection between mathematics and significant propositions[32].

This is reminiscent of Kant's account, in which the mere act of representing a number involves the repeated instantiation of an object *in intuition*, and thus the notion of number can only be abstracted from the prior unity of cognition. For Kant, the possibility of numberhood is grounded on its instantiation in empirical cognitions; there is no sense to be made of mathematics without this paradigmatic context; conversely, since the synthesis of empirical appearances is the same as the one through which we construct mathematical concepts, empirical cognition presupposes the forms of space and time with their mathematical determinations, such as those of number. For Wittgenstein, numeral signs are only conceivable insofar as they mimic features of significant propositions (namely, the repetition of operations), hence there is no sense to be made of mathematics outside the context of the use of significant propositions; conversely, since significant propositions presuppose the employment of operations, and their concatenation, there is no sense to be

as forming a hylomorphic whole.

[31] Hence the sense in which this is a definition is more akin to Kant's notion of philosophical definition as expounded in A727/B755–A732/B760.

[32] Frascolla observes that "the idea that forms of linguistic expression have an arithmetical structure is really at the cornerstone of Wittgenstein's treatment of arithmetic in the *Tractatus*" (Frascolla 1994, p. 24).

made of significant language without number. Stenlund is not wrong in stating that the mathematical notions accounted for in the *Tractatus* are best understood as linguistic signs that do not "refer" (Stenlund 2013, pp. 14–15), as we have seen that Wittgenstein did not conceive of number–signs as denoting "mathematical objects" in the way that proper names can refer to people. However, characterizing Wittgenstein's understanding of mathematics as *merely* symbolic, as opposed to ontological, might obscure the internal relation between numbers and significant propositions, whose purpose is to assert the existence of states of affairs.

We saw how both Kant's and Wittgenstein's discussions of mathematics focused on very elementary bits of the subject in their everyday employment. Potter comments that "Wittgenstein's concern was solely with explaining the application of mathematics, since he took this to be the only thing about mathematics that could be explained" (Potter 2002, p. 181). We should have, by now, developed a subtler appreciation of the author's concerns. For instance, because the paradigmatic context in which numbers arise is the repeated application of operations to significant propositions, getting clear about the "application of mathematics" is a prerequisite to developing an account of numbers in their "pure guise", such as those found in arithmetic equations. There is no such thing as a full–blown proposition (in the Tractarian sense) *about* numbers, since these are not logically extractable from the process of applying operations to significant propositions – hence the equations of mathematics are "pseudo–propositions." Rather, we *use* number–signs to mimic or display the repeatability of operations in language. This echoes Kant's belief that numbers are not objects of empirical intuition, but rather formal features of intuitions. If Wittgenstein had not stressed this aspect of numberhood, for example by thinking of numbers as being intelligible independently of significant propositions, he would have had to face the "platonist" impasse mentioned above. However, as for Kant, this issue did not arise for him. For both thinkers, the hylomorphic conception of symbols and intuition (or cognition), respectively, play a similar and essential role in reconciling the applicability of mathematics to experience with its generality.

§ 2.3 Expressions and equations of arithmetic

As mentioned above, the *Tractatus*' brief discussion of arithmetic in the 6.2s focuses on equations, the "pseudo–propositions" of mathematics. These consist of two numerical expressions connected by the equal sign. Equations mark the equivalence of two ways of combining an operation, as exemplified in 6.241. The result of applying an operation to a given proposition 2x2 times is the same as that of applying the same operation to the same proposition 4 times, i.e., the same expression can be obtained from two different combinations of repeated ordinary operations. Thus, an *arithmetic* operation characterizes the logical form of the act of bringing together numbers, i.e. repeated ordinary operations, which in turn characterize the logical form of a propositional symbol. In other words, what is displayed in an equation is a relationship *between* different forms of propositions: an equation operates *within* the level of logical syntax, by marking an equivalence in logical form. The plus sign, for instance, denotes a kind of arithmetic combination of expressions that already bear numerical form. This is similar to how, for Kant, the plus sign in a basic arithmetic expression signifies a certain mode of bringing together, in intuition, concepts of numbers, which are themselves constructed in pure intuition. Hence, in both works, distinguishing arithmetical equations amounts to drawing finer hylomorphic distinctions *within* the level of the form of intuition (Kant's pure intuition) or of the symbol (Wittgenstein's logical syntax).

These further specifications show one way in which Wittgenstein separated mathematics from logic. We saw how the logical form of a tautology can be brought out by replacing its meaningful constituents with variables; in contrast, a mathematical equation is obtained by using the appropriate variable to denote one operation in two different expressions, and then noticing the equivalence of the two logical forms. Thus, while both subjects are concerned with showing the logical syntax of significant language – the "logic of the world," (6.22) – they take different levels of abstraction within this perspective. We thus see how Wittgenstein, similarly to Kant, provides the kind of account of mathematics that his broader understanding of language demands.

Another point of continuity between the 6.2s and Kant's discussion of mathematics is the emphasis on the multiple construability of mathematical expressions. Recall how, for Kant, the same intuition of twelve fingers in succession can be seen as the image of the sum of five and seven, or of the sum of the four and eight, or of the number twelve. I take Wittgenstein to suggest a similar view in TLP 6.2323: "An equation merely marks the point of view from which I consider two expressions." The *Tractatus*' only explicit comment about mathematical expressions themselves is found at 6.231: "It is a property of '1+1+1+1' that it can be construed as '(1+1)+(1+1)'". This is strikingly reminiscent of the passage, from Kant's letter to Schultz, about the different ways of writing the number eight. I take this to show that for both Kant and Wittgenstein, substitutability and multiple construability are essential marks of numerical expressions, and that this fact should be given prominence in an account of mathematics. We saw above how to read Kant as taking this to be a reason to hold that mathematics relies on intuition. In the *Tractatus* this feature has a different, albeit related, significance. To see this, notice that 6.231 begins with a similar comment about the sameness of affirmation and double negation. In our discussion of operations, we mentioned how the fact that the same proposition can be obtained as a result of different operations corroborates the claim that operation signs do not represent, so that they must be a mark of the form of a propositional symbol and not of its content – they do not have a *Bedeutung*. Similarly, the fact that arithmetical expressions can be construed in different ways should support the claim that mathematical expressions characterize the *logical form* of propositional symbols[33]. I believe that our hylomorphic reading can help us appreciate the connection between the two ways in which the multiple construability of mathematical expressions figures in the two texts. Kant invokes the hylomorphic unity of intuitions and concepts in cognition to conclude, after contrasting the finitude of the understanding with the infinite construability of numerical expressions, that mathematics must rely on intuition. Wittgenstein invokes the hylomorphic unity of the propositional symbol to conclude that mathematical expressions, since they do not make up the significant content of the proposition, must characterize its logical form. What matters, for our purposes, is that using the language of hylomorphism helps us understand how this feature of mathematical expressions is relevant to both thinkers.

In closing my discussion of the 6.2s, I should note that the upshot of the analyses of the regions of language discussed in the 6s is not only to distinguish them from paradigmatic, assertoric discourse, but also to distinguish them from each other. We can compare this project to the one, often implicit in the first *Critique*, of distinguishing between different kinds of *a priori* judgments. This can also take the form of a hylomorphic analysis, as exemplified above by the distinction between pure general logic and transcendental logic. A crucial goal of this undertaking, for both Kant and Wittgenstein, is to distinguish each of

[33] 6.231 does, in fact, come immediately after the following sentence: "When two expressions can be substituted for one another, that characterizes their logical form".

these regions of discourse from philosophy, so as to elucidate the nature and tasks of the latter. In the first chapter of the *Doctrine of Method* (A712/B740–A738/B766), Kant embarks on an extended analysis of the differences between mathematics and philosophy, in an attempt to illuminate how it is not appropriate for the latter to imitate the mode of inquiry of the former[34]. The conception that I have been delineating should allow us to better appreciate these considerations. Mathematics and philosophy, for Kant, differ radically in their modes of cognition; for instance the former, unlike the latter, operates solely on the pure intuitive aspect of cognition. We can thus read Kant's discussion of mathematics as aimed at marking its difference from philosophy. Similarly the *Tractatus*, by considering and clarifying the nature of the pseudo–propositions of mathematics, is trying to dispel our temptation to model philosophy after this subject[35]. By getting clear about the process of abstraction through which we arrive at mathematical equations, we also recognize that what we are trying to do in philosophy is something essentially different[36]. This is yet another concern that the two authors shared.

§ 3. Concluding considerations

I could have started this paper by making a list of generic similarities between Kant's and Wittgenstein's accounts of mathematics, such as the concern with reconciling its generality with its applicability, or the idea that mathematics is not primarily characterized by a special subject matter (CPR A714/B742, TLP 6.124, 6.2). However, I believe that doing so would not have done justice to the depth of these affinities. In this paper, I have pointed to the structural parallel between Kant's hylomorphic conception of cognition [*Erkenntnis*] and

[34] He does so, for example, by meticulously distinguishing the characters and purposes of mathematical from philosophical definitions (A727/B755–A732–B760), as well as demonstrations (A734/B762–A738/B766), and he explains why *axioms* belong to mathematics and have no use in transcendental philosophy (A732/B760–A733/B761).

[35] I am borrowing this idea from (Diamond 2014, p. 154).

[36] Floyd expands on this idea: "Like Plato in the *Meno*, or Kant in the *Critique of Pure Reason*, when Wittgenstein discusses a particular logical result or a mathematical example, he is most often model- or picture-building: pursuing, through a kind of allegorical analogy, not only a better understanding of the epistemology of logic and mathematics, but also a more sophisticated understanding of the nature of philosophy conceived as an activity of self-expression and disentanglement from metaphysical confusion, for purposes of an improved mode of life" (Floyd 2005, p. 77).

Wittgenstein's hylomorphic conception of the proposition [*Satz*], both of which employ the Aristotelian notions of form and matter while stressing the priority of the unified whole over its parts. Both notions are best understood if we start by considering their paradigmatic forms: empirical cognitions in the *Critique*, and significant propositions in the *Tractatus*. In this framework, an account of mathematics must take the specific shape that we find in the two works: first, specifying the level of hylomorphic analysis with which mathematical discourse is concerned; second, filling out the details of the process of stepwise abstraction through which we arrive at the mathematical form of our object of reflection. Only by appreciating the analogously paradigmatic roles of, respectively, empirical cognition and *sinnvolle Sätze* is it possible to appreciate how central the issue of generality and applicability was to both thinkers, as well as how their conceptions of the nature of numbers present us with essentially identical solutions to the problem. Only by appreciating the similarity in the internal relations between, on the one hand, sensibility and understanding, and, on the other hand, sign and symbol, can we paraphrase Kant's statement that "The essential and distinguishing feature of pure mathematics is that it [can] only [proceed] by means of the construction of concepts [in intuition]" (Kant 1950, p. 17) with the Wittgensteinian claim that "the essential and distinguishing feature of pure mathematics is that it can only proceed by means of symbolic calculation via an adequate notation"[37]. The same exegetical work also allows us to discern further parallels between the two conceptions of mathematics, such as a similar analysis of the nature of mathematical equations and of the difference between mathematics and philosophy.

This account could also help us get clear about how the two thinkers diverge. Doing so falls beyond the scope of the present work; as a clue, however, we can observe that for Kant, mathematics deals with the form of *intuition*, which constitutes the *matter* of cognition. It differs from pure general logic in that the latter operates at the level of the concepts. For Wittgenstein, mathematics deals with the logical form of propositional *symbols*, which constitutes the linguistic *form* of a significant proposition. That is, Wittgenstein sees mathematics as operative on a similar, though distinct dimension, to that of logic. What matters here is that, for the purpose of contrasting the two conceptions of mathematics, one must first acknowledge this structural

[37] This is a reference to a sentence that was added by Wittgenstein to Ramsey's copy of the Tractatus at 6.02, quoted in (Floyd 2002, n. 14). Echoes of this claim can be found at TLP 6.2331 and 6.2341.

difference in how the two thinkers identify the perspective of mathematics. One could account for this contrast by taking, similarly to Stenlund, an historical route, stressing the difference between the mathematical traditions to which the two thinkers belonged. Alternatively, one could point to a deeper split in the natures of the two philosophical projects, namely the critiques of cognition and language. Neither account, however, can disregard the continuity that I have described.

Acknowledgements

I thank James Conant, Daniel Sutherland, and Will Small for their valuable comments on earlier drafts of this paper, as well as Juliet Floyd for her encouragement and helpful suggestions after reading the present version. I am also indebted to Zoe Chitty for her invaluable support with copyediting.

REFERENCES

BRONZO, Silver (2011). "Context, Compositionality, and Nonsense in Wittgenstein's Tractatus". In: *Beyond the Tractatus Wars: The New Wittgenstein Debate*, edited by Rupert Read and Matthew Lavery. New York: Routledge.

CONANT, James (2002). "The Method of the Tractatus". In: *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by E Reck, 374–462. New York: Oxford University Press. DOI: 10.1093/0195133269.003.0015.

CONANT, James (2016). "Why Kant Is Not a Kantian". *Philosophical Topics* vol. 44 no. 1: pp. 75–125. DOI: 10.5840/philtopics20164417.

CONANT, James (2020). "Reply to Gustafsson: Wittgenstein on the Relation of Sign to Symbol". In: *The Logical Alien*, edited by Sofia Miguens. Cambridge: Harvard University Press, pp. 863–947. DOI: 10.4159/9780674242821.

DIAMOND, Cora (2014). "The Hardness of the Soft: Wittgenstein's Early Thought About Skepticism". In *Varieties of Skepticism: Essays after Kant, Wittgenstein, and Cavell*, edited by James Conant and Andrea Kern. Berlin, Boston: De Guyter, pp. 145–181. DOI: 10.1515/9783110336795.

FLOYD, Juliet (2002). "Number and Ascriptions of Number in Wittgenstein's Tractatus". In *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by E Reck. New York: Oxford University Press, pp. 308–352. DOI: 10.1093/0195133269.003.0013.

FLOYD, Juliet (2005). "Wittgenstein on Philosophy of Logic and Mathematics". In: *The Oxford Handbook of Philosophy of Logic and Mathematics*, edited by Stewart Shapiro. Oxford: Oxford University Press, pp. 75–135. DOI: 10.1093/oxfordhb/9780195325928.001.0001.

FRASCOLLA, Pasquale (1994). *Wittgenstein's Philosophy of Mathematics*. London: Routledge. DOI: 10.4324/9780203022467.

HYLTON, Peter (2005). "Functions, Operations, and Sense in Wittgenstein's Tractatus". In: *Propositions, Functions, and Analysis: Selected Essays on Russell's Philosophy*. Oxford: Oxford University Press, pp. 138–152. DOI: 10.1093/acprof:oso/9780199286355.003.0009.

KANT, Immanuel (1950). *Prolegomena to Any Future Metaphysics*. Translated by Lewis White Back. Indianapolis: Bobbs-Merrill.

KANT, Immanuel (2007a). *Correspondence*. Edited by Arnulf Zweig.

Cambridge: Cambridge University Press.

KANT, Immanuel (2007b). *Critique of Pure Reason*. Translated by Paul Guyer and Allen Wood. New York: Cambridge University Press.

KREMER, Michael (2022). "Mathematics and Meaning in the Tractatus". *Philosophical Investigations* vol. 25 no. 3: pp. 272–303. DOI: 10.1111/1467-9205.00175.

LAND, Thomas (2014). "Spatial Representation, Magnitude and the Two Stems of Cognition". *Canadian Journal of Philosophy*, Mathematics in Kant's Critical Philosophy, vol. 44 nos. 5/6: pp. 524–550. DOI: 10.1080/00455091.2014.971688.

LOCKHART, Thomas (2016). "Prolegomena to a Proper Treatment of Mathematics in the Critique of Pure Reason". *Philosophical Topics*, Analytical Kantianism, vol. 34 no.s 1 & 2: pp. 221–281. DOI: 10.5840/philtopics2006341/29.

MARION, Matthew (1998). *Wittgenstein, Finitism, and the Philosophy of Mathematics*. New York: Oxford University Press. DOI: 10.1093/mind/110.438.501.

MONK, Ray (1991). *Ludwig Wittgenstein: The Duty of Genius*. New York: Vintage.

MOORE, Adrian (2013). "Was the Author of the Tractatus a Transcendental Idealist?". In: *Wittgenstein's Tractatus. History & Interpretation*, edited by Adrian Moore and Peter Sullivan. Oxford: Oxford University Press, pp. 239-255. DOI: 10.1093/acprof:oso/9780199665785.003.0010.

PARSONS, Charles (1983). "Kant's Philosophy of Arithmetic". In: *Mathematics in Philosophy*. Itaha: Cornell University Press, pp. 110–149.

POTTER, Michael (2002). *Reason's Nearest Kin: Philosophies of Arithmetic from Kant to Carnap*. Oxford: Oxford University Press. DOI: 10.1093/acprof:oso/9780199252619.003.0012.

SMYTH, Daniel (2014). "Infinity and Givenness: Kant on the Intuitive Origin of Spatial Representation". *Canadian Journal of Philosophy*, Mathematics in Kant's Critical Philosophy, vol. 44 nos. 5/6: pp. 524–550. DOI: 10.1080/00455091.2014.967737.

SMYTH, Daniel (2015). "Infinity and Givenness: Kant's Critical Theory of Sensibility". PhD Dissertation, Chicago: University of Chicago.

STENLUND, Sören (1990). *Language and Philosophical Problems*. London:

Routledge. DOI: 10.4324/9780203406991.

STENLUND, Sören (2013). "Wittgenstein and Symbolic Mathematics". *O Que Nos Faz Pensar* vol. 22, no. 33: pp. 7–34.

STENLUND, Sören (2014). "The Origin of Symbolic Mathematics and the End of the Science of Quantity". *Uppsala Philosophical Studies* 59 (April). DOI: 10.13140/2.1.3954.5281.

WITTGENSTEIN, Ludwig (1974). *Tractatus Logico-Philosophicus*. Edited by Brian McGuinness and David Frances Pears. 2nd ed. New York: Routledge & Kegan Paul.

WITTGENSTEIN, Ludwig (1979). *Notebooks: 1914-1916*. Translated by Gertrude Elizabeth Margaret Anscombe and Georg Henrik Von Wright. Chicago: University of Chicago Press.

WITTGENSTEIN, Ludwig (1984). *Philosophical Remarks*. Edited by Rush Rhees. Translated by Raymond Hargreaves and Roger White. Chicago: University of Chicago Press.

YOUNG, Michael (1992). "Construction, Schematism, and Imagination". In *Kant's Philosophy of Mathematics*, edited by Carl J Posy. Philadelphia: Kluwer, pp. 159–75. DOI: 10.1007/978-94-015-8046-5_7.

Mathematics, Intuition, and Language in the first *Critique* and the *Tractatus*

Proposition 6.233 from Wittgenstein's *Tractatus* has been read as a rejection of the Kantian claim that mathematics relies on intuition. Contrary to previous contributions, I develop a reading of the *Critique* and the *Tractatus* that shows how the two had a similar understanding of what kind of account mathematics calls for. I do so by focusing on a fundamental structural similarity between the Tractarian notion of *Satz* (proposition) and the Kantian one of *Erkenntnis* (cognition). I argue that we can fruitfully read much of the fundamental terminology of the two works as the result of an analysis of these two notions in terms of form and matter; moreover, both terms are best understood by considering their paradigmatic employment: empirical cognition for Kant and the significant proposition for Wittgenstein. This analysis allows us to gain a better understanding of Kantian pure intuition and Tractarian logical form of propositional symbols, thus elucidating the domains in which the two authors think mathematics operates.

Keywords: Hylomorphism · Kant · Wittgenstein · Erkenntnis · Satz.

La matemática, la intuición y el lenguaje en la primera *Crítica* y en el *Tractatus*

La proposición 6.233 del *Tractatus* de Wittgenstein se ha ido leyendo como un rechazo de la afirmación kantiana de que la matemática esté basada en la intuición. Contrariamente a contribuciones previas estoy

desarrollando una lectura de la *Crítica* y del *Tracatus* que muestra que los dos tenían un entendimiento similar acerca del tipo de explicación que la matemática requiere. Lo hago concentrándome en una similitud estructural fundamental entre la nocion de *Satz* (proposición) tractariana y la kantiana de *Erkenntnis* (conocimiento). Arguyo que se puede hacer una lectura fecunda de una gran parte de la terminología fundamental de las dos obras como resultado de un análisis de estas dos nociones en términos de forma y materia; además, ambos términos se entienden mejor si se considera su uso paradigmático: conocimiento empírico para Kant y proposiciones significativas para Wittgenstein. Este análisis nos permite obtener un mejor entendimiento de la intuición pura de Kant y de la forma lógica tractariana de símbolos proposicionales que ilumina los dominios en los cuales los dos autores piensan que la matemática está operando.
Palabras claves: Hilomorfismo · Kant · Wittgenstein · Erkenntnis · Satz.

PAOLO DEGIORGI is a PhD candidate in philosophy at Boston University. After receiving his BA from the University of Chicago in 2018, he spent three years as a PhD candidate in mathematics at the University of Illinois at Chicago. His research interests lie in the history of early analytic philosophy and German idealism.

INFORMACIÓN DE CONTACTO | CONTACT INFORMATION: Department of Philosophy and Education Sciences, Boston University, 745 Commonwealth Ave #516, 02215, Boston, MA, USA. e–mail (✉): degiorgi@bu.edu.· iD: http://orcid.org/0000-0003-2003-1579.

HISTORIA DEL ARTÍCULO | ARTICLE HISTORY
Received: *8–November–2022*; Accepted: *23–December–2022*; Published Online: *31–December–2022*

COMO CITAR ESTE ARTÍCULO | HOW TO CITE THIS ARTICLE
Degiorgi, Paolo. (2022). «Mathematics, Intuition, and Language in the first *Critique* and the *Tractatus*». *Disputatio. Philosophical Research Bulletin* 11, no. 23: pp. 137-164.